Sovereign Credit Risk and Banking Crises

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Abstract

This paper develops a structural model for the valuation of sovereign debt in which a sovereign country faces a strategic default decision under the risk of experiencing a banking crisis. The optimal default policy is governed by the trade-off between lower debt-servicing expenditures and the costs of sovereign default represented by reductions in foreign trade as well as increased financial stress for the local banking sector. The framework developed in this paper yields new insights into the interaction between sovereign risk and different financial sector characteristics such as the relative size of the banking sector within the sovereign's economy, aggregate financial sector credit risk, and bank bond holdings of public debt.

Keywords: Sovereign Debt, Credit Risk, Jump Diffusion, Endogenous Default, Jump Risk, Banking Crisis.

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1 Introduction

The link between sovereign risk and the financial system has become apparent in the latest financial crisis and plays a key role in the current sovereign debt worries in Europe. While at the onset of the crisis, culminating in Lehman Brother's bankruptcy in September 2008, fears about collapsing financial systems led to a global rise in sovereign risk, the subsequent European debt crisis also revealed significant spillovers of sovereign risk into bank credit risk, in particular through the channel of banking sector holdings of public debt. The framework developed in this paper allows to model sovereign credit risk on the basis of a fragile financial sector that exposes the sovereign to banking risk, at the same time allowing the government to credibly commit to the repayment of its debt. While being able to reproduce a variety of well-established predictions about the relation between sovereign credit spreads and macroeconomic variables, the model reveals new interdependencies between sovereign risk and different characteristics of the sovereign's domestic financial sector.

Sovereign debt differs from corporate debt in that it is not enforceable by the courts. Hence, a sovereign cannot credibly commit to hand over assets in the event of default. Since the seminal papers of Eaton & Gersovitz (1981) and Bulow & Rogoff (1989), the early sovereign debt literature emphasized capital market exclusion and trade sanctions as the main reasons why sovereigns repay their debt. While these types of sovereign default costs were clearly at play in many past default scenarios, their role in deterring sovereign default is controversial, since it is unclear whether they are economically large enough to commit a sovereign sufficiently to servicing its debt. This is particularly true for large industrialized countries that are unlikely to experience coordinated punishment by lenders. As such, recent studies focus on domestic output costs that capture direct collateral damage to the sovereign's economy, for example through the channel of bank bond holdings of government debt, as well as indirect costs that result from reputational effects causing a change in behavior of economic agents, such as foreign trading partners and investors. A series of studies point to the costs a sovereign default inflicts on the financial system. In particular Sturzenegger & Zettelmeyer (2007), Borensztein & Panizza (2009), and Panizza et al. (2009) suggest that sovereign defaults deepen economic crises through exacerbated capital flight, a deterioration of domestic banks' balance sheets, collapsing investor confidence, increased legal risk, and a higher likelihood of bank runs.

This paper develops a structural model for the valuation of sovereign debt that incorporates the above-mentioned costs into the sovereign's strategic default decision. The sovereign country endogenously determines the timing of default under the risk of experiencing a banking crisis, where the costs of default are represented by reductions in foreign trade as well as increased financial stress for the local banking sector. The sovereign's objective is to maximize the expected present value of primary expenditures defined as aggregate tax income less debt-servicing expenditures and the optimal default decision is governed by the trade-off between lower debt-servicing expenditures and the economic costs of sovereign default. The model provides a unified framework that determines sovereign credit spreads on the basis of a variety of macroeconomic and financial variables such as a sovereign's debt level, macroeconomic volatility, aggregate banking sector credit risk, financial sector size, and bank bond holdings of public debt. The paper provides theoretical results about the relation between sovereign risk and different aspects of the financial system and establishes testable predictions for future empirical research.

In particular, the model predicts that a risky financial sector exposes a sovereign to financial sector credit risk but at the same time allows the sovereign to credibly commit to the repayment of its debt. As a consequence, financial sector credit risk might affect sovereign credit risk in two ways. Depending on the specific scenario it might increase or decrease sovereign credit risk. In a similar way, the model predicts that in terms of sovereign credit risk, financial sector size is a doubled-edged sword. On the one hand, a large financial sector lowers sovereign risk as it commits the sovereign to repaying debt holders. On the other hand, a large financial sector raises the sovereign's exposure to bank credit risk. Whether a sovereign gains or loses from a large financial sector depends, besides other factors, on financial sector fragility. If the domestic banking sector is stable and the risk of a banking crisis is low, sovereigns with large financial sectors carry lower credit spreads as they are more committed to repaying their debt. However, since the same sovereigns also react more sensitively to an increase in financial sector credit risk, this situation reverses as the risk of a banking crisis becomes imminent.

The model also provides predictions about how the relation between sovereign risk and the domestic banking sector depends on other types of sovereign default costs such as reductions in foreign trade. Moreover, the model allows to study the effects of a change in bank bond holdings of public debt, for example due to the sovereign giving government debt a preferential status for meeting reserve requirements or due to international diversification of government bond holdings by banks as it has been observed in the process of financial sector integration in Europe. Finally, I discuss sovereign debt capacities, optimal debt levels, and political economy issues. Regarding the latter I focus on the effect of shortening the sovereign's planning horizon, in the sense that a sovereign decides to put more weight on economic performance that falls within its legislative period than on economic performance that lies in the distant future. Besides the above-mentioned predictions the model is able to reproduce a series of relations between sovereign risk and macroeconomic variables that are well established in the literature. In particular, sovereign risk decreases with economic growth and the sovereign's level of tax income. At the same time sovereign credit spreads increase with the level of public debt and macroeconomic volatility.

So far there exists a small but fast growing literature including Basu (2010), Bolton & Jeanne (2011), Acharya et al. (2011), and Gennaioli et al. (2012) that models the link between sovereign risk and the financial sector. In contrast to the existing literature I provide a continuous-time framework and focus on new aspects of the financial system such as the relative size of the banking sector within the sovereign's economy. I show that bank credit risk itself may change the sovereign's commitment to debt holders and discuss the relation between sovereign default costs due to banking sector fragility and other types of costs such as reductions in foreign trade. Finally, I discuss how sovereign time preferences affect optimal debt levels, sovereign credit spreads, and the sovereign's default decision.

From a technical perspective this paper is related to the strand of the sovereign risk literature that uses structural models in the tradition of Merton (1974), Black & Cox (1976), and Leland (1994) for the valuation of sovereign debt. This literature includes Gibson & Sundaresan (2001), Westphalen (2002), Andrade (2009), Jeanneret (2012b) and Jeanneret (2012a). In contrast to these studies I provide a jump diffusion framework that allows to introduce financial sector risk into the assessment of a sovereign's optimal default time and the valuation of its debt.

The paper is organized as follows. Section (2) develops a basic model of sovereign risk that entails an endogenous default decision. In this setting the sovereign's incentive to service its debt stems exclusively from its reliance on foreign trade. Section (3) extends the model by exposing the sovereign to the risk of a banking crisis and introduces financial sector vulnerability with respect to sovereign default. The calibration of the model is undertaken in section (4), which also discusses the empirical predictions about the relation between sovereign risk and a number of financial sector characteristics. Furthermore, sovereign debt capacities and the relation between sovereign credit spreads and basic macroeconomic variables are studied. A discussion of optimal debt levels and political economy issues is undertaken in the appendix. Section (5) concludes.

2 A Model of Sovereign Credit Risk

Consider an economy that is inhabited by a representative industrial firm and a representative financial firm. Together these firms produce an aggregate payout flow Z that is taxed at the national tax rate ζ. Until the sovereign defaults, its tax income flow $Y = \zeta Z$ is governed by the process:

$$dY_t = \mu Y_t dt + \sigma Y_t dW_t, \tag{1}$$

where W_t is a Brownian motion defined on the probability space $(\Omega, \mathcal{F}, F, \mathbb{P})$ and $F = \{\mathcal{F}_t : t \geq 0\}$ is the information filtration. The parameters μ and σ represent the drift and volatility of the diffusion process, respectively. Throughout the paper I assume that agents are risk-neutral and have perfect information on the state of the economy. Furthermore, I assume that there exists an instantaneous riskless interest rate r at which investors can lend and borrow freely. In the absence of sovereign default the claim to total tax income at time t is given by:

$$V_t = \mathbb{E}_t \left[\int_t^{\infty} e^{-r(s-t)} Y_s ds \right] = \frac{Y_t}{r - \mu}, \tag{2}$$

where $\mathbb{E}_t$ represents the expectation operator conditional on time t. Sovereign debt contracts are not subject to enforceable law. As such, the sovereign may choose to strategically default on its debt obligations when its tax income falls below a level Y_B. The optimal default threshold Y_B^* will be determined endogenously within the model and is based on the sovereign's trade-off between lower debt-servicing payments and the costs of sovereign default, which will be discussed below. The time of sovereign default τ can be written as:[2]

$$\tau = \inf\{t > 0 \mid Y_t \leq Y_B\}. \tag{3}$$

[2] I assume that the sovereign can default only once. However, looking at longer periods Reinhart & Rogoff (2009) find that sovereigns tend to default periodically.

2.1 The Value of Sovereign Debt

I first derive the value of sovereign debt for a given default threshold Y_B and postpone the discussion of the sovereign's optimal default barrier to the next section. I assume that the sovereign issues an infinite maturity debt contract with value D and a continuous debt service c. At the time of default the sovereign enters into renegotiation with debt holders and reduces the debt service permanently to $(1-\pi)c$, where $0 \leq \pi \leq 1$ denotes the loss rate. The value of sovereign debt D is given by:

$$D_t \quad = \quad \int_t^\infty e^{-r(s-t)} \;\; c \;\; (1-F_s) \;\; ds \quad + \quad \int_t^\infty e^{-r(s-t)} \;\; \frac{(1-\pi)c}{r} \;\; f_s \;\; ds. \tag{4}$$

The term $F_s = F(s, Y, Y_B)$ represents the cumulative probability function of the first passage time of Y to Y_B and f_s denotes the density of the first passage time. Hence, the first term in equation (4) is the expected discounted cash flow to debt holders given no default. The second term represents the expected discounted recovery when the sovereign hits the default threshold Y_B. It can be shown that the solution for the value of sovereign debt D is given by equation (5), where $-\gamma$ is the negative root of the quadratic equation $r - \epsilon\mu - \frac{1}{2}\epsilon(\epsilon-1)\sigma^2 = 0$.[3]

$$D_t = \frac{c}{r}\left[1 - \pi\left(\frac{Y_t}{Y_B}\right)^{-\gamma}\right] \tag{5}$$

2.2 The Sovereign's Default Decision

The sovereign's default decision is governed by the trade-off between the benefits and costs of default. The early literature on sovereign debt focused on capital market exclusion and trade sanctions in explaining a sovereign's incentive to service its debt. While these costs were present in past default scenarios, it is unclear whether they are economically large enough to commit a sovereign sufficiently to the repayment of its debt. In particular, Arellano (2008) shows in a reputational model that capital market exclusion alone is not sufficient to explain the low incidence of sovereign defaults observed in reality. Likewise, Arellano & Heathcote (2010) find that in a model where permanent exclusion from capital markets is the only cost of default, maximum sustainable sovereign debt levels are too low to match those observed in the real world. As such, the more recent literature on sovereign risk stresses domestic

[3]The derivation is given in the appendix in section (6.2.1).

costs of default, in particular those associated with domestic financial markets and reductions in foreign trade, the latter resulting from reputational effects instead of trade sanctions.

In this paper the costs of a sovereign default are represented by reductions in trade and a destabilization of the domestic banking sector in the form of increased risk of a banking crisis. I will introduce the former type in this section and postpone the second type to an extension of the model in the next section. The notion that sovereign default reduces external trade is well established in the literature. Rose (2005) provides evidence that debt renegotiations lead to a significant decline in bilateral trade and emphasizes that this decline should not be interpreted as resulting from trade sanctions, which have rarely been observed in past sovereign default scenarios. Borensztein & Panizza (2010) find strong support for the hypothesis that sovereign default hurts the more export-oriented industries disproportionately and Rose & Spiegel (2004) provide indirect evidence by showing that higher levels of international trade are associated with higher levels of bilateral lending.

I assume that the sovereign's objective is to maximize the expected present value of future primary expenditure flows $Y - c$. Primary expenditures represent the part of the sovereign's tax income that can be used to finance public services and political programs and hence offer a device to attract voters. If the sovereign defaults, its debt service falls to $(1 - \pi)c$ but at the same time the sovereign's economy suffers a permanent output loss due to reductions in foreign trade. This output loss reduces the sovereign's tax income flow to $(1 - \phi)Y$, where $0 < \phi < 1$ represents the sovereign's trade-openness and thereby its reliance on external trade.[4] In the basic model I will assume that the sovereign puts equal weight on future tax income flows. Later in section (6.1) in the appendix I will address political economy issues by allowing the sovereign to put greater weight on early tax income flows that fall in the government's legislative period than on tax income flows that lie in the distant future. The expected present value of the sovereign's primary expenditures is given by:[5]

[4]In principle this output loss can capture any type of sovereign default cost. Because of the significance of the foreign trade channel in the sovereign debt literature, I interpret ϕ as stemming from reductions in external trade.

[5]It is assumed that the costs of sovereign default satisfy: $(1 - \phi)Y > (1 - \pi)c$. Hence, after default the sovereign's tax income is sufficient to pay the reduced debt service.

$$
\begin{aligned}
E_t \;&=\; \int_t^{\infty} e^{-r(s-t)} \;\; (Y_s^e - c) \;\; (1 - F_s) \;\; ds && (6)\\
&+\; \int_t^{\infty} e^{-r(s-t)} \;\; \left[\frac{(1-\phi)Y_B}{r-\mu} - \frac{(1-\pi)c}{r}\right] \;\; f_s \;\; ds,
\end{aligned}
$$

where $Y_s^e = \mathbb{E}_t[Y_s \mid Y_{\min,s} \geq Y_B]$ denotes the expectation of Y conditional on not previously hitting the default boundary. The first integral in equation (6) represents the expected present value of primary expenditures given no default. The second integral is the expected present value of primary expenditures if the sovereign hits the default threshold Y_B. The solution for the value of primary expenditures E is given by:[6]

$$
E_t = \frac{Y_t}{r-\mu} - \frac{\phi Y_B}{r-\mu}\left(\frac{Y_t}{Y_B}\right)^{-\gamma} - D_t. \qquad (7)
$$

The first two terms represent the expected present value of the sovereign's tax income flow, while D_t is the value of sovereign debt defined in equation (5). The solution for the optimal default boundary Y_B^* that maximizes E_t is determined by the smooth pasting condition (8) and is given by equation (9):

$$
\begin{aligned}
\frac{\partial E}{\partial Y}_{|Y=Y_B^*} \;&=\; \frac{(1-\phi)}{r-\mu}, && (8)\\
Y_B^* \;&=\; \frac{\gamma(r-\mu)\pi c}{(1+\gamma)r\phi}. && (9)
\end{aligned}
$$

As such, the optimal default threshold Y_B^* is increasing in the sovereign's debt level, captured by the continuous debt service c, and decreasing in the costs of default, represented by the degree of trade-openness ϕ. Besides strategic default, the sovereign might also default due to inability to pay, which occurs when its tax income flow Y falls below its debt service c. As a consequence, the minimum value for the default boundary is given by $Y_{B,\min} = c$. Plugging Y_B^* into equation (5) and accounting for the minimum barrier $Y_{B,\min}$ gives the final solution for the value of sovereign debt and the sovereign credit spread cs:

[6]The derivation is given in the appendix in section (6.2.2).

$$D_t^* = \frac{c}{r}\left[1-\pi\left(\frac{Y_t}{\max\left[\frac{\gamma(r-\mu)\pi c}{(1+\gamma)r\phi}, c\right]}\right)^{-\gamma}\right], \tag{10}$$

$$cs = \frac{c}{D_t^*} - r. \tag{11}$$

3 Sovereign Credit Risk and Banking Crises

In this section I introduce the eventuality of default of the representative financial firm[7] causing a large drop in the sovereign's tax income flow and triggering an exogenous default of the sovereign. At the same time I extend the model by a second type of sovereign default costs, which is increased financial stress for the domestic banking sector resulting in an increase in the representative financial firm's default risk.

Several recent studies emphasize the negative impact of sovereign default on the domestic financial sector. De Paoli et al. (2006) provide evidence that recent major default crises have been associated with banking crises and that the output costs are particularly large when sovereign default is combined with a financial crisis. Sturzenegger & Zettelmeyer (2007) work out specific channels through which sovereign default worsens economic crises and emphasize bank runs and exacerbated capital flights. Borensztein & Panizza (2009) argue that sovereign default may put the domestic financial system under significant stress due to uncertainty about creditor rights, a negative effect on the balance sheets of banks that hold government bonds, and a collapse in confidence that might lead to bank runs. They find that sovereign defaults drastically increase the probability of a banking crisis. Panizza et al. (2009) argue that large sovereign defaults may trigger reputational spillovers that undermine confidence in the government and increase legal risk for investors and depositors. Furthermore, Gennaioli et al. (2012) show that sovereign defaults are followed by large reductions in private credit flows and that these reductions are more severe in countries where the domestic banking sector holds large amounts of sovereign debt. Reinhart & Rogoff (2010) empirically document the co-occurrence of private and public financial crises. Using data on sovereign bond holdings of European banks obtained from the 2010 European bank stress tests, Acharya et al. (2011) provide evidence that CDS spreads of banks co-move with those of sovereigns in accordance with banks' holdings of government bonds. Finally, Bolton & Jeanne

[7] Henceforth, I refer to the representative financial firm also as financial sector or banking sector.

(2011), Acharya et al. (2011), and Gennaioli et al. (2012) show that in a number of countries, covering advanced economies as well as emerging markets, banks hold significant amounts of public debt, stressing the adverse effects a sovereign default may have on the balance sheets of the domestic banking sector.

I define default of the representative financial firm as the first arrival time of a Poisson process with constant intensity λ. Hence, in each time interval the representative financial firm may suffer a credit event and default with intensity λ causing a loss of kY in the sovereign's tax income, where $0 < k < 1$ captures the relative size of the representative financial firm within the sovereign's economy. Until default, the dynamics of the sovereign's tax income flow are now governed by a simple Poisson jump-diffusion process:

$$dY_t \;=\; \mu Y_t dt + \sigma Y_t dW - kY_t dq_t, \tag{12}$$

where W_t is a Brownian motion and q_t is a standard Poisson process defined on the probability space $(\Omega, \mathcal{F}, F, \mathbb{P})$ with $F = \{\mathcal{F}_t : t \geq 0\}$ being the information filtration. The parameters μ and σ are constant and represent the drift and volatility of the jump-diffusion, respectively. Consequently, in each time period Δt the probability that Y jumps to $(1-k)Y_t$, given survival to the beginning of this period, is approximately given by $\lambda\Delta t$ for small Δt. The cumulative probability of no jump prior to time s is given by $e^{-\lambda s}$.

To provide closed form solutions for the sovereign credit spread and the optimal default boundary, I make the assumption that in case of default of the representative financial firm the sovereign always defaults on a fraction on its debt, where the size of this fraction depends on the amount of tax income lost due to this event. Hence, sovereign default may occur either abruptly due to the occurrence of a jump (jump to default) or gradually due to diffusion to an endogenous threshold Y_B (diffusion to default). While a jump to default is an exogenous event to the sovereign, the timing of diffusion to default is endogenous and strategically chosen by the sovereign. The analysis is restricted to a single sovereign default with the exception that after a diffusion to default there is still the possibility of a subsequent default of the representative financial firm and hence a jump to a second default. Consequently, the sovereign can default up to a maximum of two times.[8] The different default scenarios of the sovereign are given below:

[8]Neglecting the possibility of a jump after diffusion to default would enable the sovereign to protect itself from such an event by choosing a high default threshold Y_B.

$$\text{default scenario:} \begin{cases} \text{no default} \\ \text{jump to default} \\ \text{diffusion to default} & \begin{cases} \text{no subsequent default} \\ \text{jump to second default} \end{cases} \end{cases} \tag{13}$$

The times of the first and second default τ_1 and τ_2 can be written as:

$$\begin{aligned} \tau_1 &= \inf\{t > 0 \mid Y_t \leq Y_B \vee dq_t = 1\}, \\ \tau_2 &= \inf\{t > \tau_1 \mid dq_t = 1 \wedge dq_{\tau_1} = 0\}. \end{aligned} \tag{14}$$

As discussed above, the literature has suggested a number of channels through which sovereign default may destabilize the domestic banking sector, such as negative balance sheet effects via bank bond holdings of public debt, reputation effects, and increased investor uncertainty about creditor rights. In particular, Borensztein & Panizza (2009) provide empirical evidence that sovereign default leads to a significant increase in the probability of a subsequent banking crisis. I account for these findings by assuming that after a sovereign default the risk of a subsequent default of the representative financial firm increases to $\tilde{\lambda} > \lambda$. Hence, the framework allows to incorporate the costs a sovereign default inflicts on its domestic financial sector, in the form of increased credit risk, into the sovereign's default decision. The increased likelihood of default of the representative financial firm after sovereign default creates a threat that increases the sovereign's commitment to repay its debt. The size of this threat depends, besides other factors, on two key characteristics: the size of the financial firm captured by the parameter k and its credit risk prior to sovereign default λ. The next section derives the value of sovereign debt and the sovereign's optimal default decision in the extended framework.

3.1 The Value of Sovereign Debt

As discussed in section (2.1) the sovereign issues an infinite maturity debt contract with value D and a continuous debt service c. At the time of default, the sovereign permanently reduces its debt service to $(1 - \pi_u)c$, where $0 < \pi_u < 1$ represents the loss rate in the default scenario u:

$$\pi_u = \begin{cases} \pi_d & \text{diffusion to default} \\ \pi_{j,1} & \text{jump to default} \\ \pi_{j,2} & \text{jump to default subsequent to diffusion to default} \end{cases} \tag{15}$$

The value of sovereign debt D is given by:

$$\begin{aligned} D_t &= \int_t^\infty e^{-r(s-t)} \; c \; (1-F_s)e^{-\lambda(s-t)} \; ds \\ &+ \int_t^\infty e^{-r(s-t)} \; D_{d,\tau_d} \; f_s e^{-\lambda(s-t)} \; ds \\ &+ \int_t^\infty e^{-r(s-t)} \; \frac{(1-\pi_{j,1})c}{r} \; (1-F_s)\lambda e^{-\lambda(s-t)} \; ds. \end{aligned} \tag{16}$$

The first integral in equation (16) is the expected present value of cash flows to debt holders given no default, the second integral represents the expected present value of cash flows if the sovereign hits the default barrier Y_B, and the third integral is the expected present value of cash flows to debt holders if the sovereign jumps to default due to a default of the representative financial firm. As in section (2.1), the term $F_s = F(s, Y, Y_B)$ represents the cumulative probability function of the first passage time of Y to Y_B. The variable f_s denotes the density of the first passage time and the cumulative probability of no jump to default until time s is given by $e^{-\lambda(s-t)}$. The variable D_{d,τ_d} denotes the value of sovereign debt at the time of diffusion to default and is given by:

$$\begin{aligned} D_{d,\tau_d} &= \int_{\tau_d}^\infty e^{-r(s-\tau_d)} \; (1-\pi_d)c \; e^{-\widetilde{\lambda}(s-\tau_d)} \; ds \\ &+ \int_{\tau_d}^\infty e^{-r(s-\tau_d)} \; \frac{(1-\pi_d)(1-\pi_{j,2})c}{r} \; \widetilde{\lambda} e^{-\widetilde{\lambda}(s-\tau_d)} \; ds, \end{aligned} \tag{17}$$

where $\tau_d = \inf\{t > 0 \mid Y_t \leq Y_B\}$ is the random time when the sovereign diffuses to default. The first integral in equation (17) represents the expected present value of reduced coupon payments $(1-\pi_d)c$ to debt holders after the default threshold Y_B has been hit given that no subsequent jump to default occurs. The second integral is the expected present value of cash flows to debt holders if the representative financial firm defaults, subsequent to the sovereign hitting the default barrier. In this case the sovereign defaults a second time, reducing its debt service to $(1-\pi_d)(1-\pi_{j,2})c$. Note that equation (17) uses $\widetilde{\lambda} = \xi\lambda$, with $\xi > 1$, instead of λ to account for the fact that sovereign default increases the probability of a subsequent default of the representative financial firm. Using Itô's lemma for Poisson processes[9] it can be shown that for a fixed default barrier Y_B the solution for the value of sovereign debt D is given by:[10]

$$D_t = \frac{cr + (1-\pi_{j,1})c\lambda}{r(r+\lambda)}\left[1 - \left(\frac{Y_t}{Y_B}\right)^{-\gamma}\right] + D_{d,\tau_d}\left(\frac{Y_t}{Y_B}\right)^{-\gamma}, \tag{18}$$

$$D_{d,\tau_d} = \frac{(1-\pi_d)cr + (1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{r(r+\widetilde{\lambda})},$$

where $-\gamma$ is the negative root of the quadratic equation $(r+\lambda)-\epsilon\mu-\frac{1}{2}\epsilon(\epsilon-1)\sigma^2 = 0$. The next section endogenizes the sovereign's default threshold.

3.2 The Sovereign's Default Decision

In line with section (2.2) the sovereign's default decision is governed by the trade-off between lower debt-servicing payments and the costs of default, which are now represented by an increase in the default intensity of the representative financial firm from $\widetilde{\lambda}$ to λ, as well as an immediate loss in the sovereign's tax income flow due to reductions in foreign trade captured by the parameter ϕ_u, where u denotes the default scenario as given in (15).[11] The sovereign's objective is to maximize the expected present value of primary expenditures given by:

$$\begin{aligned} E_t &= \int_t^\infty e^{-r(s-t)} \; (Y_s^e - c) \; (1-F_s)e^{-\lambda(s-t)} \; ds \\ &+ \int_t^\infty e^{-r(s-t)} \; E_{d,\tau_d} \; f_s e^{-\lambda(s-t)} \; ds \\ &+ \int_t^\infty e^{-r(s-t)} \left[\frac{(1-k)(1-\phi_{j,1})Y_s^e}{r-\mu} - \frac{(1-\pi_{j,1})c}{r}\right] (1-F_s)\lambda e^{-\lambda(s-t)} \; ds, \end{aligned} \tag{19}$$

[9]See Dixit & Pindyck (1994).

[10]The derivation of equation (18) is given in section (6.3).

[11]Without affecting the main results I will assume throughout the paper that $\phi_d = \phi_{j,1} = \phi_{j,2}$.

where again $Y_s^e = \mathbb{E}_t[Y_s \mid Y_{\min,s} \geq Y_B]$ denotes the expectation of Y conditional on not previously hitting the default boundary. In line with equation (16) the first integral in equation (19) is the expected present value of the sovereign's primary expenditures given no default, the second integral represents the expected present value of primary expenditures if the sovereign hits the default barrier Y_B, and the third integral is the expected present value of primary expenditures when the sovereign jumps to default. The variable E_{d,τ_d} denotes the value of primary expenditures at the time of diffusion to default:

$$
\begin{aligned}
E_{d,\tau_d} &= \int_{\tau_d}^{\infty} e^{-r(s-\tau_d)} \; [(1-\phi_d)Y_B - (1-\pi_d)c] \; e^{-\widetilde{\lambda}(s-\tau_d)} \; ds \\
&+ \int_{\tau_d}^{\infty} e^{-r(s-\tau_d)} \; \frac{(1-k)(1-\phi_d)(1-\phi_{j2})Y_B}{r-\mu} \; \widetilde{\lambda} e^{-\widetilde{\lambda}(s-\tau_d)} \; ds \\
&- \int_{\tau_d}^{\infty} e^{-r(s-\tau_d)} \; \frac{(1-\pi_d)(1-\pi_{j,2})c}{r} \; \widetilde{\lambda} e^{-\widetilde{\lambda}(s-\tau_d)} \; ds,
\end{aligned} \tag{20}
$$

where $\tau_d = \inf\{t > 0 \mid Y_t \leq Y_B\}$ is the random time when the sovereign hits the default barrier. The first line in equation (20) is the expected present value of primary expenditures after the default threshold Y_B has been hit given that no subsequent jump to default occurs, while the second and third lines represent the expected present value of primary expenditures if the sovereign jumps to default subsequent to hitting the default barrier. For a given default threshold Y_B the solution for the value of primary expenditures E_t is given by:[12]

$$
\begin{aligned}
E_t &= \overline{A}Y_t + \overline{B}Y_B \left(\frac{Y_t}{Y_B}\right)^{-\gamma} - D_t, \qquad (21) \\
\overline{A} &= \frac{1}{r+\lambda-\mu} + \frac{(1-k)(1-\phi_{j,1})\lambda}{(r-\mu)(r+\lambda-\mu)}, \\
\overline{B} &= \frac{(1-\phi_d)}{r+\widetilde{\lambda}-\mu} - \frac{1}{r+\lambda-\mu} + \frac{(1-k)(1-\phi_d)(1-\phi_{j,2})\widetilde{\lambda}}{(r-\mu)(r+\widetilde{\lambda}-\mu)} - \frac{(1-k)(1-\phi_{j,1})\lambda}{(r-\mu)(r+\lambda-\mu)}.
\end{aligned}
$$

Again, $-\gamma$ is the negative root of the quadratic equation $(r+\lambda)-\epsilon\mu-\frac{1}{2}\epsilon(\epsilon-1)\sigma^2 = 0$ and D_t is the value of sovereign debt defined in equation (18). The optimal default threshold Y_B^* that maximizes E satisfies the smooth-pasting condition:

[12] The derivation of equation (21) can be found in the appendix in section (6.4).

$$\frac{\partial E}{\partial Y}_{|Y=Y_B^*} = \frac{\partial E_d}{\partial Y}_{|Y=Y_B^*}, \tag{22}$$

and is given by:[13]

$$\begin{aligned}
Y_B^* &= \max\left[\frac{\gamma \overline{C}}{\overline{F} + \gamma \overline{B} - \overline{A}}, c\right], \qquad (23)\\
\overline{C} &= \frac{(1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{r(r+\widetilde{\lambda})} + \frac{(1-\pi_d)c}{r+\widetilde{\lambda}} - \frac{c}{r+\lambda} - \frac{(1-\pi_{j,1})c\lambda}{r(r+\lambda)},\\
\overline{F} &= \frac{(1-\phi_d)}{r+\widetilde{\lambda}-\mu} + \frac{(1-k)(1-\phi_d)(1-\phi_{j,2})\widetilde{\lambda}}{(r-\mu)(r+\widetilde{\lambda}-\mu)}.
\end{aligned}$$

In equation (23) it was accounted for that due to inability to pay, the minimum default boundary equals $Y^*_{B,\min} = c$. Plugging Y_B^* into the value of sovereign debt given by equation (18) yields the final solution for the sovereign debt contract and for the sovereign's credit spread:

$$\begin{aligned}
D_t^* &= \frac{cr + (1-\pi_{j,1})c\lambda}{r(r+\lambda)}\left[1 - \left(\frac{Y_t}{\max\left[\frac{\gamma\overline{C}}{\overline{F}+\gamma\overline{B}-\overline{A}}, c\right]}\right)^{-\gamma}\right] + \qquad (24)\\
&+ \frac{(1-\pi_d)cr + (1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{r(r+\widetilde{\lambda})}\left(\frac{Y_t}{\max\left[\frac{\gamma\overline{C}}{\overline{F}+\gamma\overline{B}-\overline{A}}, c\right]}\right)^{-\gamma},\\
cs &= \frac{c}{D_t^*} - r,
\end{aligned}$$

where $\overline{A}$, $\overline{B}$, $\overline{C}$, and $\overline{F}$ are given by equations (21) and (23). Plugging Y_B^* into equation (21) yields the final solution for the value of primary expenditures:

$$E_t^* = \overline{A}Y_t + \overline{B}\left(\max\left[\frac{\gamma\overline{C}}{\overline{F}+\gamma\overline{B}-\overline{A}}, c\right]\right)\left(\frac{Y_t}{\max\left[\frac{\gamma\overline{C}}{\overline{F}+\gamma\overline{B}-\overline{A}}, c\right]}\right)^{-\gamma} - D_t^*. \tag{25}$$

[13]The derivation is given in the appendix in section (6.4.3).

4 Empirical Predictions

This section discusses the model's empirical predictions about the sovereign's default decision and credit spread. To determine these quantities a number of parameters must be chosen, which are summarized in table (1). I calibrate the model such that it roughly reflects an industrialized country in a distressed macroeconomic environment. I define a base case scenario as well as deviations from this standard setting, representing sovereigns with low and high degrees of trade-openness as well as small and large financial sectors.

I set the risk free rate r to 4.4%, in line with the average German Bund 10 Year bond yield in the period between 1996 and 2010. The initial level of sovereign debt income Y is normalized to 100. It is assumed that the sovereign has a national tax ratio of 27.1%[14] and a debt-to-GDP ratio of 87%. Both numbers reflect average ratios in the euro area according to OECD (2009) and IMF (2010) statistics. Assuming a coupon rate equal to 5%, this yields a debt service c of approximately 16.[15]

The parameter ϕ_u captures the immediate decrease in the sovereign's tax income flow upon default. I interpret this loss in tax income as resulting from reductions in foreign trade and assume that ϕ_u increases with the sovereign's trade-openness and hence its reliance on external trade. To get a range of possible values for the parameter ϕ_u, I refer to Borensztein & Panizza (2009) and Sturzenegger & Zettelmeyer (2007), who estimate total output costs of sovereign default. In particular, these studies find that sovereign default is associated with a decrease in growth that ranges between 0.5 and 2 percentage points per year. Given an average length of sovereign default episodes in the period between 1991 and 2004 of 3.5 years[16] this yields total output losses in the range of between 1.8 and 7 percent. In the standard setting, I choose $\phi_u = 3\%$, somewhere at the lower range of the estimated output costs discussed above.[17] I define a sovereign with a low degree of trade-openness by $\phi_u^{low} = 2\%$ and a sovereign with a high degree of trade openness by $\phi_u^{high} = 4\%$.

According to Moody's estimates for the period between 1998 and 2008[18], I set the loss rate for the first sovereign default, either diffusion to default or jump to default, to $\pi_d = \pi_{j,1} = 68\%$. I assume that a possible second default is more costly for the sovereign in the sense that for the same amount of output costs, the sovereign can

[14] Excluding social security contributions.
[15] $c = (100 * 0.87 * 0.05)/(100 * 0.271) = 0.1605$.
[16] See Borensztein & Panizza (2009).
[17] I assume that ϕ_u is the same for each default scenario u.
[18] See Moodys (2011).

reduce its debt service only by a lesser fraction $\pi_{j,2} < \pi_d = \pi_{j,1}$. This assumption is supposed to capture the fact that with higher debt reductions, debt restructuring becomes increasingly difficult. In particular Bolton & Jeanne (2007) point out that different classes of sovereign debt, such as widely-dispersed public debt vs. private debt, are associated with different restructuring costs and that the sovereign may first default on those debt classes where restructuring costs are low. Hence, repeated or higher debt reductions might require the sovereign to engage in more complicated debt restructurings. As such, I set the loss rate in case of a second default $\pi_{j,2}$ to 20%, somewhere at the lower range of the historical loss rates reported by Moodys (2011).

To proxy the parameter λ, I calculate market implied default intensities from weekly 5Y EUR denominated CDS spreads of European banks that were included in the 2010 EU-wide stress tests. I study a pre-crisis period ranging from January 2004 to December 2006 as well as a crisis period from July 2008 to April 2012, covering the rise in global sovereign credit risk after Lehman Brother's bankruptcy in September 2008 as well as the current European debt crisis. For each country and each week I average over the CDS spreads of the respective banks. Then for each country I take the average, the minimum value, and the maximum value over the sample period. The numbers are reported in table (3).[19]

Under the assumption of a recovery rate equal to the market convention of 40%, I find average implied default intensities in the range between 0.02 and 0.08 over the cross-section of countries. Over the whole country sample I find an average and median implied default intensity of 0.04 and 0.03, respectively. Naturally, these numbers provide only a rough proxy as they represent risk-neutral default intensities carrying, besides other factors, risk premia, liquidity risk, counterparty risk, and legal risk. I set the parameter λ to 0.03 in the standard scenario. Based on Borensztein & Panizza (2009), who find that sovereign default significantly increases the probability of a subsequent banking crisis, I set $\widetilde{\lambda} = 2\lambda$.[20]

Later, I will discuss how the sovereign's credit spread and its optimal default decision change with the banking sector's vulnerability with respect to sovereign default represented by the parameter $\xi = \frac{\widetilde{\lambda}}{\lambda}$. Cecchetti et al. (2009) study the output costs of 40 banking crises since 1980 and find average GDP losses of 18.4%. In line with

[19]The methodology of calculating implied default intensities is discussed in section (6.7).

[20]Borensztein & Panizza (2009) find for a sample of 149 countries and the period from 1975 to 2000 that conditional on having a sovereign default in year t or $t-1$, the probability of a banking crisis in year t more than quadruples. Since in my model the increase in the default intensity is a permanent one, I use a much more conservative specification.

their findings I set the jump size k to 18% in the standard scenario. I define a sovereign with a large representative financial firm by having a k^{high} of 28% and a sovereign with a small representative financial firm by k^{low} equal to 8%. I assume that a sovereign with a large financial sector and hence higher losses in its tax income flow upon default of the representative financial firm also defaults on a greater fraction of its debt. Accordingly, I set $\pi_{j,1}^{high} > \pi_{j,1} > \pi_{j,1}^{low}$ and $\pi_{j,2}^{high} > \pi_{j,2} > \pi_{j,2}^{low}$. The parameter values are given in table (1). Finally, I set the growth rate of the sovereign's tax income flow μ to 3% and the volatility σ to 20% in the base case scenario.

4.1 Sovereign Credit Risk and the Financial Sector

The notion that there is a link between bank credit risk and sovereign credit risk is strongly supported by empirical evidence and has become particularly important since the latest financial crisis. Figure (16) shows the Thomson Reuters 5Y CDS Index for banks and sovereigns in Europe during the crisis period. While it is well established that bank and sovereign credit spreads have become increasingly integrated since the start of the crisis, the direction of causality between sovereign risk and bank credit risk changed as the crisis evolved. While at the beginning of the crisis, culminating in Lehman Brother's default, fears about collapsing financial systems led to a global rise in sovereign risk, the subsequent European debt crisis revealed considerable spillovers of sovereign risk into banking risk.[21] Besides government guarantees and rescue packages, an important reason for sovereign risk affecting bank credit risk is the fact that banks typically hold significant amounts of government bonds.[22] The framework developed in this paper provides a way of analyzing what this two-way relation between sovereign and bank credit risk means for a sovereign's timing of default and hence its credit spread.

Figure (1) illustrates the relation between sovereign credit risk and the credit risk of the financial firm represented by the jump risk λ. An increase in λ has two competing effects on sovereign risk. On the one hand, an increase in λ raises sovereign credit risk because it implies a higher probability that the representative financial firm defaults. On the other hand, an increase in λ lowers sovereign credit risk as it also implies a higher level of vulnerability of the financial firm with respect to sovereign default. This increases the sovereign's commitment to repay its debt and lowers its optimal default boundary as seen in figure (29). If λ is small, then the second effect dominates

[21]See Ejsing & Lemke (2009), Bolton & Jeanne (2011), and Acharya et al. (2011).

[22]See Bolton & Jeanne (2011), Acharya et al. (2011), and Gennaioli et al. (2012).

the first and the sovereign's credit spread decreases in financial sector credit risk. However, as λ becomes larger the first effect dominates the second and the sovereign's credit spread eventually increases in λ. In other words, in the extreme case where λ goes to zero, the representative financial firm becomes riskless, the sovereign loses an important commitment device and sovereign credit risk increases. In figure (1) the minimum credit spread corresponds to a λ of 0.005, which is roughly equivalent to an annual CDS spread of 30 basis points. Hence, for the base case scenario sovereign credit spreads decrease in financial sector credit risk until $\lambda \approx 0.005$. Most European banks studied in table (3) show CDS spreads and default intensities well below this number in the pre-crisis period.

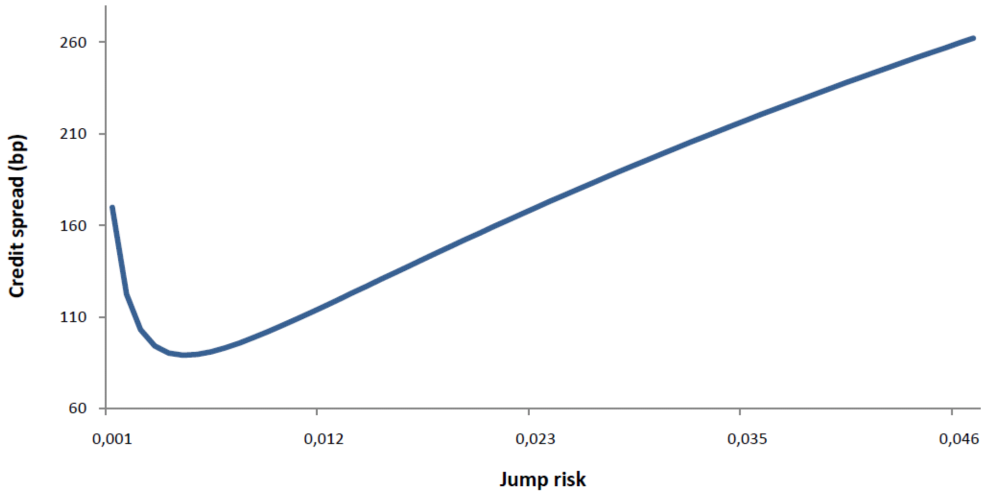

Figure 1: Credit spread vs. jump risk λ

Next I study how the size of the financial firm affects the relation between sovereign credit spreads and the parameter λ. Figure (2) illustrates this relation for two sovereigns, both with a low degree of trade-openness. As defined in the appendix in table (2), one sovereign exhibits a small financial sector while the other sovereign exhibits a large financial sector. The main insight from figure (2) is that, in terms of sovereign credit risk, financial sector size is a double-edged sword. On the one hand, a large financial sector lowers sovereign credit spreads by increasing the sovereign's commitment to service its debt. Since default of the representative financial firm inflicts higher output losses for a sovereign with a large financial sector and a sovereign default increases the likelihood of such an event, a sovereign with a large financial sector is less inclined to strategically default on its debt and hence exhibits a lower optimal default boundary as illustrated by figure (17). On the other hand, a sovereign with a large financial sector suffers higher economic losses if a banking crisis actually occurs. Whether a sovereign gains or loses from a large financial sector in terms of credit risk depends on the state of the representative financial firm. If λ is low, a sovereign with a large financial sector carries a lower credit spread as it is more committed to repaying its debt. However, the same sovereign also reacts more sensitively to an increase in λ. Hence, as λ becomes large and the risk of a banking crisis imminent, the situation reverses and the sovereign with a large financial sector carries a higher spread.

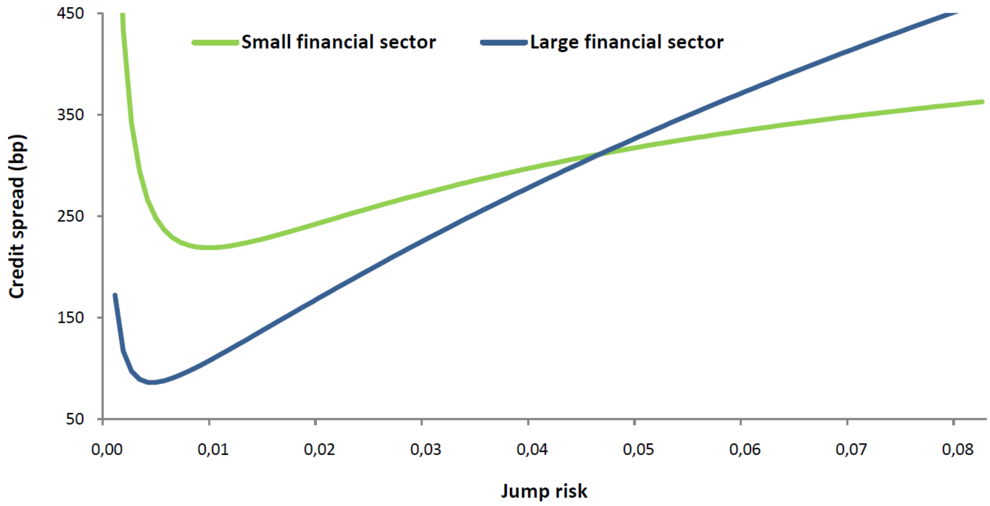

Figure 2: Sovereign credit spread vs. jump risk λ

Figure (2) shows a hump at lower levels of the parameter λ because, as discussed above, bank credit risk itself increases the sovereign's commitment to debt holders and decreases the optimal default boundary. The predictions of figure (2) are in line with the empirical findings of Gerlach et al. (2010), who study sovereign bond spreads in the euro area for the period between 1999 and 2009. The authors find that when aggregate risk, as measured by the US corporate bond spread, is low, sovereign countries with large financial sectors, as measured by total bank assets to GDP, carry lower credit spreads than countries with small financial sectors. At the same time, credit spreads of countries with large financial sectors react more sensitively when aggregate risk increases. The model developed in this paper provides a possible theoretical foundation for these findings.

The model also predicts that the possible gain of a large financial sector decreases with the sovereign country's reliance on foreign trade. Figure (3) illustrates the same scenario as in figure (2) but with a parameter ϕ_u for illustrative purposes set to the very high value of 10%. In this scenario a large financial sector has no additional effect on the sovereign's commitment to repaying its debt because due to the high degree of trade-openness the sovereign is already fully committed in the sense that its optimal default boundary is already at its minimum value $Y^*_{B,\min}$. As a consequence, the sovereign with a large financial sector exhibits a higher credit spread for all values of λ.

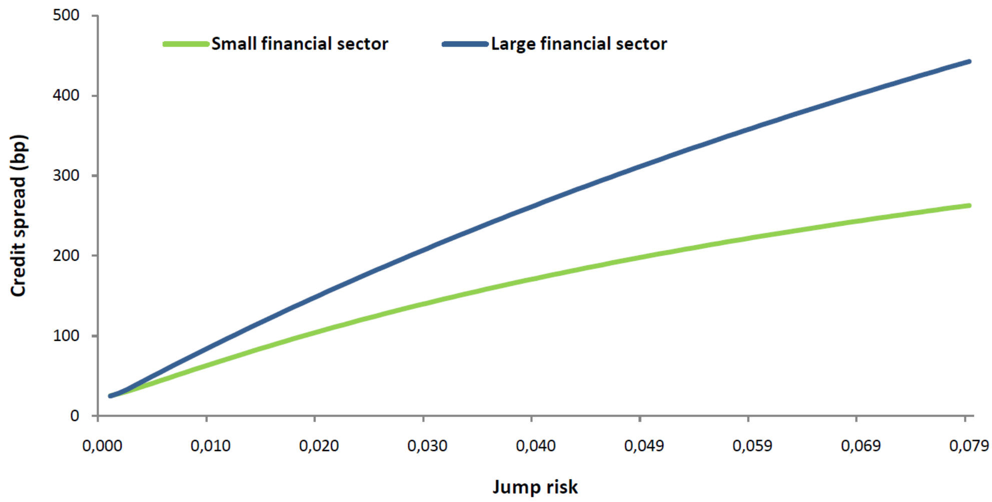

Figure 3: Sovereign credit spread vs. jump risk λ

Figure (4) demonstrates the relation between financial sector size and sovereign credit spreads for different degrees of trade-openness.[23] Similar to bank credit risk, the size of the sovereign's financial sector affects sovereign risk in two ways. First, a large financial sector reduces sovereign risk by increasing the sovereign's commitment to service its debt. Second, a large financial sector increases sovereign risk by inflicting higher output losses in the event that the representative financial firm actually defaults. As figure (4) demonstrates, the first effect dominates when k is small, leading to a negative relation between financial sector size and sovereign credit risk for low and medium degrees of trade-openness. However, as k rises the second effect increasingly outweighs the first one, leading to a positive relation between sovereign risk and financial sector size. As before, the overall effect on sovereign credit spreads depends on the degree of trade-openness. For a sovereign that relies little on foreign trade, an increase in financial sector size has a strong commitment effect leading to a sharp decline in sovereign credit risk. In contrast, a sovereign that relies heavily on foreign trade is fully committed to repaying its debt even without financial sector risk. Figure (19) shows the corresponding optimal default boundaries. A result of

[23]The parametrization of the link between loss rates in case of a jump to default ($\pi_{j,1}$ and $\pi_{j,2}$) and the jump size k is given in the appendix in equation (75).

figure (4) is that for economically and politically powerful countries that cannot be threatened by traditional sources of sovereign default costs such as reductions in foreign trade, a large financial sector is a way of committing to the repayment of public debt.

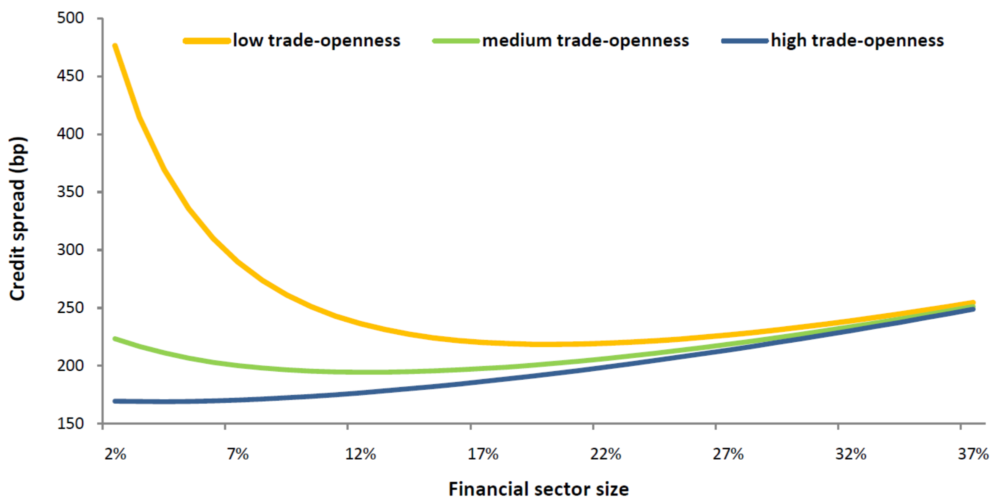

Figure 4: Credit spread vs. financial sector size k

I summarize the following key predictions about the interaction between sovereign credit risk and different aspects of the sovereign's financial sector:

1. *In normal times, when the risk of a banking crisis is low, sovereign countries with large financial sectors carry lower credit spreads than countries with small financial sectors due to a commitment effect: sovereigns with large financial sectors are less inclined to strategically default on their debt because a default would destabilize their financial systems. However, in countries with large financial sectors, sovereign credit risk also reacts more sensitively to an increase in financial sector credit risk, because if a banking crisis occurs, which might be independent and prior to sovereign default, the sovereign with a large financial sector suffers higher economic losses. As a result, if a banking crisis becomes imminent sovereign countries with large financial sectors carry higher credit spreads than countries with small financial sectors.*

2. *The commitment effect that arises from a large financial sector decreases with other sources of sovereign default costs such as the degree of trade-openness. Hence, sovereign countries that rely heavily on foreign trade benefit less from a large financial sector in terms of sovereign risk.*

3. *Financial sector credit risk itself affects the sovereign's commitment to repay its debt. For low levels of financial sector credit risk an increase in the risk of a banking crisis lowers sovereign credit spreads, since it increases the sovereign's commitment to debt holders as the financial sector becomes more vulnerable to sovereign default. As such, in the extreme case where the sovereign's banking sector becomes default-free the sovereign loses an important commitment device and sovereign risk increases.*

4.2 Bank bond holdings of public debt

As discussed in section (3.2), it is assumed that sovereign default raises the default risk of the representative financial firm from λ to $\widetilde{\lambda} = \xi\lambda$, with $\xi > 1$. The size of ξ and hence the financial sector's vulnerability to a sovereign default depends, besides other factors, on the exposure of domestic banks to government debt. Acharya et al. (2011) find that banks participating in the 2010 European stress tests hold on average about one sixth of their risk-weighted assets in sovereign bonds and that about 69% of these bonds are issued by the country in which a bank is headquartered. Bolton & Jeanne (2011) estimate that in the euro area the average share of government debt held by domestic banks amounted to roughly 15% by the end of 2009. Gennaioli et al. (2012) study bank holdings of public debt for several emerging markets and find bond holdings of up to 50% of total bank assets. Gennaioli et al. (2012) also point out that governments have several ways to increase bank holdings of public debt, for example by giving government debt a preferential status for meeting reserve requirements. Hence, one can think of the parameter ξ as being subject to public policy measures. As such, by granting public debt a preferential status for meeting reserve requirements, a sovereign might increase financial sector vulnerability to sovereign default. That a sovereign might indeed do such a thing is illustrated in figure (5), which shows the relation between sovereign credit spreads and the parameter ξ for different levels of trade-openness.

Higher holdings of public debt make the domestic financial sector more vulnerable to sovereign default, thereby committing the sovereign to repaying its debt and decreasing the sovereign's credit spread. Again the size of this commitment effect depends negatively on the degree of trade-openness. Figure (20) shows the optimal default boundaries corresponding to figure (5).

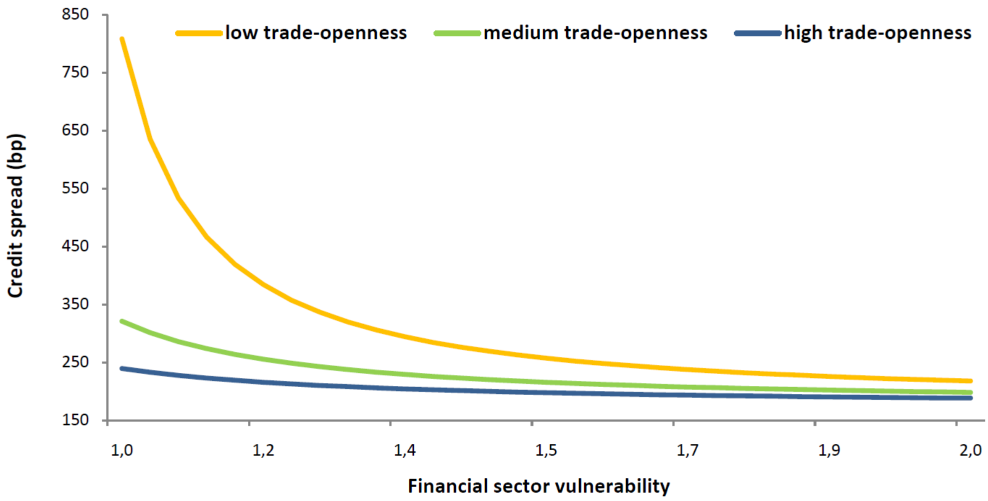

Figure 5: Credit spread vs. financial sector vulnerability ξ

This finding can also be related to the international diversification of government bond holdings by European banks that followed the establishment of the European Monetary Union. Bolton & Jeanne (2011) show that within Europe the share of foreign government debt in total government debt held by banks is large, reaching 70% in some countries. A key result of Bolton & Jeanne (2011) is that in the absence of fiscal integration, financial integration (in the form of international diversification of government bonds held by banks) benefits safe countries. Financial integration hurts safe countries as they are now open to sovereign risk contagion from risky countries. However, this effect is outweighed by the fact that banks in risky sovereign countries are now willing to pay a premium for holding safe-country government debt. The model developed in the present paper suggests that there is an additional cost with the international diversification of bank holdings of public debt. As banks diversify their holdings, they decrease their exposure to domestic government bonds, thereby reducing the financial sector's vulnerability to default of the local sovereign. This reduces the sovereign's commitment to service its debt and may increase its credit spread.

Note that there is a subtle difference between the effects of increasing financial sector size k as outlined in figure (4) and increasing the banking sector's vulnerability to a sovereign default ξ as outlined in figure (5). While in both cases the sovereign decreases its optimal default boundary Y_B^* due to higher commitment, an increase in financial sector size is more costly for the sovereign in the sense that it also increases the sovereign's losses if a banking crisis occurs prior to default of the sovereign. In contrast, an increase in the parameter ξ only affects the sovereign's expected losses after default. For this reason, the sovereign's credit spread is eventually increasing in financial size k while an increase in the parameter ξ can increase the sovereign spread only after the sovereign has reduced its optimal default boundary to its minimum value $Y^*_{B,\min}$. I summarize the empirical predictions of this section in the following way:

4. *An increase in financial sector holdings of sovereign debt – which might be initiated by granting government debt a preferential status for meeting reserve requirements – raises the banking sector's vulnerability to sovereign default, thereby committing the sovereign to the repayment of debt. Conversely, international diversification of government debt held by domestic banks increases the banking sector's resistance to sovereign default, thereby decreasing the sovereign's commitment to bond holders and increasing sovereign credit spreads.*

4.3 Sovereign debt capacity

In this section I discuss the effect of an increase in financial sector vulnerability on the sovereign's debt capacity. Figure (21) shows the value of sovereign debt D^*_t given by equation (24) as a function of the sovereign's coupon payment c. On the one hand, an increase in the coupon payment increases the sovereign's debt value due to higher debt-servicing payments. On the other hand, it also makes sovereign debt riskier by raising the optimal default boundary Y^*_B. The sovereign's maximum debt value $D^{\max}_t$ is achieved at $c^{\max}$, which will be referred to as the sovereign's debt capacity:

$$\begin{aligned} c^{\max} &= \arg\max_c \frac{cr + (1-\pi_{j,1})c\lambda}{r(r+\lambda)} \left[1 - \left(\frac{Y_t}{\max\left[\frac{\gamma\overline{C}}{\overline{F}+\gamma\overline{B}-\overline{A}}, c\right]} \right)^{-\gamma} \right] + \\ &+ \frac{(1-\pi_d)cr + (1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{r(r+\widetilde{\lambda})} \left(\frac{Y_t}{\max\left[\frac{\gamma\overline{C}}{\overline{F}+\gamma\overline{B}-\overline{A}}, c\right]} \right)^{-\gamma}, \end{aligned} \tag{26}$$

where $\overline{A}$, $\overline{B}$, $\overline{C}$, and $\overline{F}$ are given by equations (21) and (23). I numerically solve for $c^{\max}$ using grid search. Figure (6) shows debt capacities $c^{\max}$ for different degrees of financial sector vulnerability ξ. As expected, an increase in ξ raises the sovereign's commitment to debt holders and increases the sovereign's maximum attainable debt value $D_t^{\max}$.

Figures (22) and (23) demonstrate the effect of trade-openness and financial sector size on the sovereign's debt capacity using the base case calibration given in tables (1) and (2). I summarize the key finding of this section in the following way:

5. *An increase in financial sector holdings of sovereign debt raises financial sector vulnerability with respect to sovereign default. This increases the sovereign's commitment to debt holders and thereby the sovereign's debt capacity.*

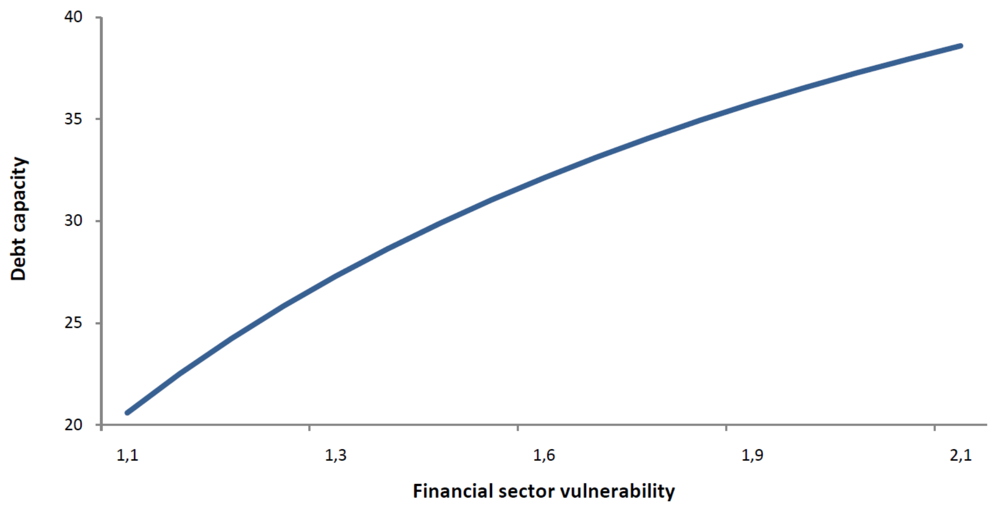

Figure 6: Debt capacity vs. financial sector vulnerability ξ

4.4 Other Comparative Statics

In addition to the empirical predictions discussed above, the model is able to reproduce a series of standard predictions about sovereign risk that are well established in the literature. In particular, sovereign credit spreads decrease in economic growth and the sovereign's tax base. At the same time credit spreads increase in macroeconomic volatility, the level of public debt, and the degree of trade-openness.

The comparative statics of sovereign credit spreads are illustrated in figures (7) to (15). Figures (7) and (8) show that credit spreads decrease with the level of tax income Y and its growth rate μ. Hence, given a constant national tax rate, the sovereign's credit spread decreases with aggregate output and economic growth.

This prediction is in line with several empirical studies, such as Cantor & Packer (1996) and Catao & Sutton (2002).

Figure (9) illustrates that higher macroeconomic volatility σ raises credit spreads. In accordance with this prediction, Catao & Sutton (2002) and Catao & Kapur (2004) provide empirical evidence that higher macroeconomic volatility increases sovereign risk. Note that the sovereign's credit spread remains positive even when volatility σ goes to zero due to the possibility that the sovereign jumps to default. Figure (13) shows the relation between credit spreads and macroeconomic volatility for sovereigns with low, medium, and high degrees of trade-openness as defined in table (2). A sovereign with a high degree of trade-openness, and hence high losses in its tax income flow upon default, faces a lower optimal default threshold and hence a lower credit spread. Figure (13) shows that the credit spreads of all three types of sovereigns meet at some point and then increase at the same rate. This feature arises from the fact that there exists a natural minimum default boundary $Y^*_{B,\min}$ which equals the debt service c. As illustrated by figure (14), the sovereign reduces its default boundary to this minimum value when macroeconomic volatility is high enough. At this point strategic default is ruled out and the sovereign defaults only due to inability to pay. Hence, from this point onwards all three types of sovereigns exhibit the same credit spread.

Figure (10) demonstrates the effect of an increase in the debt service c on sovereign credit spreads. As expected, sovereign credit risk increases with the level of public debt, which is strongly supported by many empirical studies such as Hilscher & Nosbusch (2010) and Gerlach et al. (2010). Figure (11) presents the relation between sovereign credit spreads and the degree of trade-openness captured by the parameter ϕ_u. Higher default costs due to greater reductions in foreign trade increase the sovereign's commitment to service its debt and hence decrease the optimal default threshold. Through this channel a higher degree of trade-openness decreases sovereign credit risk.

5 Conclusion

This paper develops a structural model for the valuation of sovereign debt in which a sovereign country faces a strategic default decision under the risk of jumping to default due to a banking crisis. The sovereign endogenously determines the timing of default by maximizing the present value of primary expenditures, defined as aggregate tax income less debt-servicing expenditures. The optimal default policy is governed by the trade-off between lower debt-servicing expenditures and the costs of sovereign default represented by reductions in foreign trade and a destabilization of the domestic financial system resulting in an increased probability of a banking crisis. While the model is capable of reproducing a variety of basic predictions about the relation between sovereign credit risk and macroeconomic variables such as output growth, macroeconomic volatility, and the level of public debt, it yields new insights into the interaction between sovereign risk and different aspects of the financial sector.

In particular, the model predicts that a large financial sector affects sovereign risk in two ways. On the one hand, it lowers sovereign credit risk by committing the sovereign to servicing its debt. On the other hand, it raises sovereign risk by increasing the potential losses in the event of a banking crisis. Which effect dominates depends on the quality of the financial system in terms of aggregate bank credit risk as well as on the sovereign country's trade-openness. It turns out that in terms of sovereign credit risk a large financial sector is an asset to the sovereign in normal times. However, this situation reverses as the risk of a banking crisis becomes imminent. Furthermore, the model predicts that bank credit risk itself imposes a commitment effect on the sovereign, as increased financial sector fragility makes the sovereign less inclined to default on its debt. In the same way, an increase in bank bond holdings of public debt raises the banking sectors vulnerability with respect to sovereign default and commits the sovereign to repaying bond holders. Such a change in the banking sector exposures to government debt might result from the government giving sovereign debt a preferential status for meeting reserve requirements or from international diversification of government bond holdings by banks, as it has been observed in the process of financial integration in Europe. Finally, this paper provides predictions on sovereign debt capacities and a discussion of changing a sovereign's time preferences with respect to future tax income flows that is undertaken in the appendix.

6 Appendix

6.1 Optimal Debt Level and Political Economy

So far it has been assumed that the government faces an exogenous debt level and has a long term view when optimizing the expected present value of future primary expenditures in the sense that it weights primary expenditure flows equally along all time periods. This section discusses the implications of relaxing both assumptions.

Assume that a government starts with zero debt and has the option to issue a debt contract with continuous debt service c. Taking on debt increases the sovereign's budget today by D_t^* but requires debt-servicing expenditures of c in the future. The sovereign chooses c such that the value of sovereign debt plus the value of primary expenditures $D_t^* + E_t^*$, as given by equations (24) and (25), is maximized. Abstracting away from the possibility of sovereign default, the total value of the sovereign is given by:[24]

$$D_t^{\text{nodef}} + E_t^{\text{nodef}} = \frac{c}{r} + \frac{[(r-\mu) + (1-k)\lambda]\,Y}{(r+\lambda-\mu)(r-\mu)} - \frac{c}{r} = \frac{[(r-\mu) + (1-k)\lambda]\,Y}{(r+\lambda-\mu)(r-\mu)}. \quad (27)$$

Hence, under the assumption that the sovereign cannot default on its debt, the total value of the sovereign $D_t^{\text{nodef}} + E_t^{\text{nodef}}$ is independent of the level of debt. This is a form of the Barro-Ricardo equivalence proposition[25]. In the absence of sovereign default, the value of the sovereign is independent of its debt level. As shown in figure (24), allowing for the possibility of sovereign default and introducing economic costs of default, the total value of the sovereign $D_t^* + E_t^*$ is decreasing in the sovereign's debt service c. Hence, so far the model provides no explanation why the sovereign takes up debt in the first place. While there may be various reasons why a sovereign should take on debt, I will now focus on political economy issues. In particular, I will assume that a sovereign gives higher weight to early primary expenditure flows that might fall in the government's legislative period and lower weight to primary expenditure flows that lie in the distant future. Technically, I assume that the sovereign discounts future primary expenditure flows at a rate $r + \eta$, where $\eta > 0$. The parameter η can be interpreted as a political preference rate that is negatively related to the length of the sovereign's planning horizon. When the sovereign chooses

[24] The derivation is given in the appendix in section (6.4.4).

[25] See Barro (1974).

its optimal default boundary and its optimal debt service, it now maximizes E_t^ρ and $D_t^{*\rho} + E_t^{*\rho}$, respectively:

$$\begin{aligned} E_t^\rho &= \overline{A}^\rho Y_t + \overline{B}^\rho Y_B \left(\frac{Y_t}{Y_B}\right)^{-\delta} - D_t, \qquad (28) \\ \overline{A}^\rho &= \frac{1}{r+\eta+\lambda-\mu} + \frac{(1-k)(1-\phi_{j,1})\lambda}{(r+\eta-\mu)(r+\eta+\lambda-\mu)}, \\ \overline{B}^\rho &= \frac{(1-\phi_d)}{r+\eta+\widetilde{\lambda}-\mu} - \frac{1}{r+\eta+\lambda-\mu} + \frac{(1-k)(1-\phi_d)(1-\phi_{j,2})\widetilde{\lambda}}{(r+\eta-\mu)(r+\widetilde{\lambda}-\mu)} \\ &- \frac{(1-k)(1-\phi_{j,1})\lambda}{(r+\eta-\mu)(r+\lambda-\mu)}. \end{aligned}$$

The parameter $-\delta$ is the negative root of the quadratic equation $(r+\eta+\lambda) - \epsilon\mu - \frac{1}{2}\epsilon(\epsilon-1)\sigma^2 = 0$ and $D_t^{*\rho}$ as well as $E_t^{*\rho}$ are obtained by plugging the optimal default boundary $Y_B^{*\rho}$ given by equation (29) into equations (18) and (28).

$$\begin{aligned} Y_B^{*\rho} &= \max\left[\frac{\delta\overline{C}^\rho}{\overline{F}^\rho + \delta\overline{B}^\rho - \overline{A}^\rho}, c\right], \qquad (29) \\ \overline{C}^\rho &= \frac{(1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{(r+\eta)(r+\eta+\widetilde{\lambda})} + \frac{(1-\pi_d)c}{r+\eta+\widetilde{\lambda}} - \frac{c}{r+\eta+\lambda} - \frac{(1-\pi_{j,1})c\lambda}{(r+\eta)(r+\eta+\lambda)}, \\ \overline{F}^\rho &= \frac{(1-\phi_d)}{r+\eta+\widetilde{\lambda}-\mu} + \frac{(1-k)(1-\phi_d)(1-\phi_{j,2})\widetilde{\lambda}}{(r+\eta-\widetilde{\mu})(r+\eta+\widetilde{\lambda}-\mu)}. \end{aligned}$$

Expressions E_t^ρ and $Y_B^{*\rho}$ are equivalent to those given by equations (21) and (23), except that the parameter r was replaced by $r+\eta$. I numerically optimize $D_t^{*\rho} + E_t^{*\rho}$ with respect to c. As demonstrated in figure (25) a positive political preference rate η induces the sovereign to take on debt and hence leads to a positive optimal debt service $c^* > 0$. If a sovereign puts greater weight on today's primary expenditures than on later flows, the sovereign will attach a higher value to funds that are obtained through the issuance of debt, and hence are available immediately, than to funds lost in the future due to higher debt-servicing payments. Note that the optimal debt service c^* eventually decreases in η because a more shortsighted sovereign is also more risky as an increase in η raises the sovereign's optimal default boundary $Y_B^{*\rho}$ and hence decreases the value of sovereign debt $D_t^{*\rho}$. These features are illustrated in figures (26) and (28). As such, a more shortsighted sovereign also carries a higher credit spread, as demonstrated in figure (27).

6.2 A Model of Sovereign Credit Risk

6.2.1 The Value of Sovereign Debt

In section (2.1) the value of sovereign debt D is given by:

$$D_t = \int_t^\infty e^{-r(s-t)} \ c \ (1 - F_s) \ ds + \int_t^\infty e^{-r(s-t)} \ \frac{(1-\pi)c}{r} \ f_s \ ds. \tag{30}$$

No arbitrage implies that D must satisfy the following differential equation:

$$rDdt = cdt + \mathbb{E}[dD]. \tag{31}$$

Using Itô's lemma equation (31) can be written as:

$$rD = c + \mu Y \frac{\partial D}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 D}{\partial Y^2}. \tag{32}$$

The general solution to equation (32) is:

$$D = AY^{-\gamma} + BY^{-\delta} + \frac{c}{r}, \tag{33}$$

where

$$\begin{aligned} \gamma &= \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right], \qquad (34) \\ \delta &= \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right], \end{aligned}$$

and A and B are constants which are determined by the boundary conditions:

$$\lim_{Y_t \to \infty} D_t = \frac{c}{r}, \tag{35}$$

$$\lim_{Y_t \to Y_B} D_t = \frac{(1-\pi)c}{r}.$$

Since δ is negative, the boundary conditions require that $B = 0$. Otherwise, $BY^{-\delta}$ explodes as $Y \to \infty$. The constant A is determined by the value-matching condition:

$$AY_B^{-\gamma} + \frac{c}{r} = \frac{(1-\pi)c}{r}. \tag{36}$$

Hence,

$$D_t = \frac{c}{r}\left[1 - \pi\left(\frac{Y_t}{Y_B}\right)^{-\gamma}\right]. \tag{37}$$

6.2.2 The Value of Primary Expenditures

In section (2.2) the value of the sovereign's primary expenditures E is given by:

$$\begin{aligned} E_t &= \int_t^\infty e^{-r(s-t)} \ (Y_s^e - c) \ (1 - F_s) \ ds \\ &+ \int_t^\infty e^{-r(s-t)} \ \left[\frac{(1-\phi)Y_B}{r-\mu} - \frac{(1-\pi)c}{r}\right] \ f_s \ ds, \end{aligned} \tag{38}$$

No arbitrage and Itô's lemma imply that E must satisfy the following differential equation:

$$rE = (Y - c) + \mu Y \frac{\partial E}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 E}{\partial Y^2}. \tag{39}$$

The general solution to equation (39) is:

$$D = CY^{-\gamma} + DY^{-\delta} + \frac{Y}{r-\mu} - \frac{c}{r}, \tag{40}$$

where $D = 0$ by arguments equivalent to those in section (6.2.1). The constant C is determined by the value-matching condition:

$$CY_B^{-\gamma} + \frac{Y_B}{r-\mu} - \frac{c}{r} = \frac{(1-\phi)Y_B}{r-\mu} - \frac{(1-\pi)c}{r}. \tag{41}$$

Hence,

$$E_t = \frac{Y}{r-\mu} - \frac{\phi Y_B}{r-\mu}\left(\frac{Y}{Y_B}\right)^{-\gamma} - \frac{c}{r}\left[1 - \pi\left(\frac{Y_t}{Y_B}\right)^{-\gamma}\right]. \tag{42}$$

6.3 Sovereign Credit Risk and Banking Crises

6.3.1 The Value of Sovereign Debt after Diffusion to Default

In section (3.1) the value of sovereign debt after diffusion to default is given by:

$$\begin{aligned} D_{d,\tau_d} &= \int_{\tau_d}^{\infty} e^{-r(s-\tau_d)}\ (1-\pi_d)c\ e^{-\widetilde{\lambda}(s-\tau_d)}\ ds \\ &+ \int_{\tau_d}^{\infty} e^{-r(s-\tau_d)}\ \frac{(1-\pi_d)(1-\pi_{j,2})c}{r}\ \widetilde{\lambda} e^{-\widetilde{\lambda}(s-\tau_d)}\ ds. \end{aligned} \tag{43}$$

No arbitrage implies that D_d must satisfy the following differential equation:

$$rD_d dt = (1-\pi_d)cdt + \mathbb{E}[dD_d]. \tag{44}$$

Using Itô's lemma for Poisson processes:

$$\mathbb{E}[dD_d] = \left(\mu Y \frac{\partial D_d}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 D_d}{\partial Y^2}\right) dt + \widetilde{\lambda}\left(\frac{(1-\pi_d)(1-\pi_{j,2})c}{r} - D_d\right) dt, \tag{45}$$

the value of sovereign debt satisfies:

$$(r+\widetilde{\lambda})D_d = (1-\pi_d)c + \frac{(1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{r} + \mu Y \frac{\partial D_d}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 D_d}{\partial Y^2}. \tag{46}$$

The general solution to equation (46) is:

$$D_d = AY^{-\gamma} + BY^{-\delta} + \frac{(1-\pi_d)c}{r+\widetilde{\lambda}} + \frac{(1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{r(r+\widetilde{\lambda})}, \tag{47}$$

where,

$$\gamma = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2(r+\widetilde{\lambda})\sigma^2}\,\right], \tag{48}$$

$$\delta = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2(r+\widetilde{\lambda})\sigma^2}\,\right].$$

Since the analysis is restricted to a single diffusion to default, and hence after the sovereign diffuses to default there can only be one more jump to default, boundary conditions require that $A = 0$ and $B = 0$. Hence, the solution is given by:

$$D_{d,\tau_d} = \frac{(1-\pi_d)cr + (1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{r(r+\widetilde{\lambda})}. \tag{49}$$

6.3.2 The Value of Sovereign Debt before Diffusion to Default

In section (3.1) the value of sovereign debt before diffusion to default is given by:

$$\begin{aligned} D_t &= \int_t^{\infty} e^{-r(s-t)}\ c\ (1-F_s)e^{-\lambda(s-t)}\ ds \\ &+ \int_t^{\infty} e^{-r(s-t)}\ D_{d,\tau_d}\ f_s e^{-\lambda(s-t)}\ ds \\ &+ \int_t^{\infty} e^{-r(s-t)}\ \frac{(1-\pi_{j,1})c}{r}\ (1-F_s)\lambda e^{-\lambda(s-t)}\ ds. \end{aligned} \tag{50}$$

No arbitrage implies that D_d must satisfy the following differential equation:

$$rDdt = cdt + \mathbb{E}[dD]. \tag{51}$$

Using Itô's lemma for Poisson processes:

$$\mathbb{E}[dD] = \left(\mu Y\frac{\partial D}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 D}{\partial Y^2}\right)dt + \lambda\left(\frac{(1-\pi_{j,1})c}{r} - D\right)dt, \tag{52}$$

the value of sovereign debt satisfies:

$$(r+\lambda)D = c + \frac{(1-\pi_{j,1})c\lambda}{r} + \mu Y \frac{\partial D}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 D}{\partial Y^2}. \tag{53}$$

The general solution to equation (53) is:

$$D = AY^{-\gamma} + BY^{-\delta} + \frac{c}{r+\lambda} + \frac{(1-\pi_{j,1})c\lambda}{r(r+\lambda)}. \tag{54}$$

Since δ is negative, boundary conditions require that $B = 0$. Otherwise, $BY^{-\delta}$ explodes as $Y \to \infty$. The constant A is determined by the value-matching condition:

$$AY_B^{-\gamma} + \frac{c}{r+\lambda} + \frac{(1-\pi_{j,1})c\lambda}{r(r+\lambda)} = D_{d,\tau_d}. \tag{55}$$

It follows that the solution for the value of sovereign debt D is given by:

$$D_t = \frac{cr + (1-\pi_{j,1})c\lambda}{r(r+\lambda)} \left[1 - \left(\frac{Y}{Y_B}\right)^{-\gamma}\right] + D_{d,\tau_d} \left(\frac{Y}{Y_B}\right)^{-\gamma}. \tag{56}$$

6.4 The Value of Primary Expenditures

6.4.1 The Value of Primary Expenditures after Diffusion to Default

In section (3.2) the value of primary expenditures E is given by:

$$\begin{aligned} E_{d,\tau_d} &= \int_{\tau_d}^{\infty} e^{-r(s-\tau_d)} \; [(1-\phi_d)Y_B - (1-\pi_d)c] \; e^{-\widetilde{\lambda}(s-\tau_d)} \; ds \\ &+ \int_{\tau_d}^{\infty} e^{-r(s-\tau_d)} \; \frac{(1-k)(1-\phi_d)(1-\phi_{j2})Y_B}{r-\mu} \; \widetilde{\lambda} e^{-\widetilde{\lambda}(s-\tau_d)} \\ &- \int_{\tau_d}^{\infty} e^{-r(s-\tau_d)} \; \frac{(1-\pi_d)(1-\pi_{j,2})c}{r} \; \widetilde{\lambda} e^{-\widetilde{\lambda}(s-\tau_d)} \; ds. \end{aligned} \tag{57}$$

No arbitrage implies that E_d must satisfy the following differential equation:

$$rE_d dt = [(1-\phi_d)Y_B - (1-\pi_d)]cdt + \mathbb{E}[dE_d]. \tag{58}$$

Using Itô's lemma for Poisson processes:

$$\begin{aligned}
\mathbb{E}[dE_d] &= \left(\mu Y \frac{\partial E_d}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 E_d}{\partial Y^2}\right) dt \qquad (59)\\
&+ \widetilde{\lambda}\left(\frac{(1-k)(1-\phi_d)(1-\phi_{j,2})Y_B}{r-\mu} - \frac{(1-\pi_d)(1-\pi_{j,2})c}{r} - E_d\right) dt,
\end{aligned}$$

the value of primary expenditures satisfies:

$$\begin{aligned}
(r+\widetilde{\lambda})E_d &= (1-\phi_d)Y_B + \frac{(1-k)(1-\phi_d)(1-\phi_{j,2})Y_B\widetilde{\lambda}}{r-\mu} \qquad (60)\\
&- (1-\pi_d)]c - \frac{(1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{r}\\
&+ \mu Y \frac{\partial E_d}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 E_d}{\partial Y^2}.
\end{aligned}$$

The general solution to equation (60) is:

$$\begin{aligned}
E_d &= AY^{-\gamma} + BY^{-\delta} + \frac{(1-\phi_d)Y_B}{r+\widetilde{\lambda}-\mu} + \frac{(1-k)(1-\phi_d)(1-\phi_{j,2})Y_B\widetilde{\lambda}}{(r-\mu)(r+\widetilde{\lambda}-\mu)} \qquad (61)\\
&- \frac{(1-\pi_d)c}{r+\widetilde{\lambda}} - \frac{(1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{r(r+\widetilde{\lambda})},
\end{aligned}$$

Since the analysis is restricted to a single diffusion to default, boundary conditions require that $A = 0$ and $B = 0$ and the solution is given by:

$$E_d = \frac{(1-\phi_d)Y_B}{r+\widetilde{\lambda}-\mu} + \frac{(1-k)(1-\phi_d)(1-\phi_{j,2})Y_B\widetilde{\lambda}}{(r-\mu)(r+\widetilde{\lambda}-\mu)} - D_d. \qquad (62)$$

6.4.2 The Value of Primary Expenditures before Diffusion to Default

In section (3.1) the value of primary expenditures before diffusion to default is given by:

$$
\begin{aligned}
E_t &= \int_t^{\infty} e^{-r(s-t)} \;(Y_s^e - c)\; (1-F_s)e^{-\lambda(s-t)} ds \qquad (63)\\
&+ \int_t^{\infty} e^{-r(s-t)} \;E_{d,\tau_d}\; f_s e^{-\lambda(s-t)}\; ds \\
&+ \int_t^{\infty} e^{-r(s-t)} \;\left[\frac{(1-k)(1-\phi_{j,1})Y_s^e}{r-\mu} - \frac{(1-\pi_{j,1})c}{r}\right]\; (1-F_s)\lambda e^{-\lambda(s-t)}\; ds \\
Y_s^e &= \mathbb{E}[Y_s \mid Y_{\min,s} \geq Y_B].
\end{aligned}
$$

No arbitrage implies that E must satisfy the following differential equation:

$$
rEdt = (Y-c)dt + \mathbb{E}[dE_d]. \qquad (64)
$$

Using Itô's lemma for Poisson processes:

$$
\begin{aligned}
\mathbb{E}[dE] &= \left(\mu Y \frac{\partial E}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 E}{\partial Y^2}\right) \qquad (65)\\
&+ \lambda\left(\frac{(1-k)(1-\phi_{j,1})Y}{r-\mu} - \frac{(1-\pi_{j,1})c}{r} - E\right) dt,
\end{aligned}
$$

the value of primary expenditures satisfies:

$$
\begin{aligned}
(r+\lambda)E &= Y + \frac{(1-k)(1-\phi_{j,1})\lambda Y}{r-\mu} \qquad (66)\\
&- c - \frac{(1-\pi_{j,1})c\lambda}{r} + \mu Y \frac{\partial E}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 E}{\partial Y^2}.
\end{aligned}
$$

The general solution to equation (66) is:

$$
\begin{aligned}
E &= AY^{-\gamma} + BY^{-\delta} + \frac{Y}{r+\lambda-\mu} + \frac{(1-k)(1-\phi_{j,1})\lambda Y}{(r-\mu)(r+\lambda-\mu)} \qquad (67)\\
&- \frac{c}{r+\lambda} - \frac{(1-\pi_{j,1})c\lambda}{r(r+\lambda)}.
\end{aligned}
$$

Since δ is negative, boundary conditions require that $B = 0$. Otherwise, $BY^{-\delta}$ explodes as $Y \to \infty$. The constant A is determined by the value-matching condition:

$$\begin{aligned} E_d &= AY_B^{-\gamma} + \frac{Y_B}{r+\lambda-\mu} + \frac{(1-k)(1-\phi_{j,1})\lambda Y_B}{(r-\mu)(r+\lambda-\mu)} \\ &- \frac{c}{r+\lambda} - \frac{(1-\pi_{j,1})c\lambda}{r(r+\lambda)}. \end{aligned} \tag{68}$$

Hence, the solution is given by:

$$E = \overline{A}Y + \overline{B}Y_B\left(\frac{Y}{Y_B}\right)^{-\gamma} - D, \tag{69}$$

$$\overline{A} = \frac{1}{r+\lambda-\mu} + \frac{(1-k)(1-\phi_{j,1})\lambda}{(r-\mu)(r+\lambda-\mu)},$$

$$\overline{B} = \frac{(1-\phi_d)}{r+\widetilde{\lambda}-\mu} - \frac{1}{r+\lambda-\mu} + \frac{(1-k)(1-\phi_d)(1-\phi_{j,2})\widetilde{\lambda}}{(r-\mu)(r+\widetilde{\lambda}-\mu)} - \frac{(1-k)(1-\phi_{j,1})\lambda}{(r-\mu)(r+\lambda-\mu)}.$$

6.4.3 Optimal Default Boundary

The optimal default threshold Y_B^* that maximizes the expected present value of primary expenditures E satisfies the smooth-pasting condition:

$$\frac{\partial E}{\partial Y}_{|Y=Y_B^*} = \frac{\partial E_d}{\partial Y}_{|Y=Y_B^*}. \tag{70}$$

From equations (62) and (69) the optimal default threshold is given by:

$$Y_B^* = \frac{\gamma\overline{C}}{\overline{F}+\gamma\overline{B}-\overline{A}}, \tag{71}$$

$$\overline{C} = \frac{(1-\pi_d)(1-\pi_{j,2})c\widetilde{\lambda}}{r(r+\widetilde{\lambda})} + \frac{(1-\pi_d)c}{r+\widetilde{\lambda}} - \frac{c}{r+\lambda} - \frac{(1-\pi_{j,1})c\lambda}{r(r+\lambda)}, \tag{72}$$

$$\overline{F} = \frac{(1-\phi_d)}{r+\widetilde{\lambda}-\mu} + \frac{(1-k)(1-\phi_d)(1-\phi_{j,2})\widetilde{\lambda}}{(r-\mu)(r+\widetilde{\lambda}-\mu)}.$$

6.4.4 Sovereign Value without Default

Section (6.1) discusses the sovereign's debt D_t^{nodef} and the value of primary expenditures E_t^{nodef} in the case of no sovereign default. The derivation of these expressions is equivalent to those discussed above:

$$
\begin{aligned}
D_t^{\text{nodef}} &= \int_t^{\infty} e^{-r(s-t)} \;\; c \;\; e^{-\lambda(s-t)} \;\; ds \qquad (73) \\
&= \frac{S}{r}
\end{aligned}
$$

$$
\begin{aligned}
E_t^{\text{nodef}} &= \int_t^{\infty} e^{-r(s-t)} \;\; (Y_s - c) \;\; e^{-\lambda(s-t)} ds \qquad (74) \\
&+ \int_t^{\infty} e^{-r(s-t)} \;\; \left(\frac{(1-k)Y}{r-\mu} - \frac{S}{r} \right) \;\; \lambda e^{-\lambda(s-t)} \;\; ds \\
&= \frac{[(r-\mu) + (1-k)\lambda]\, Y}{(r+\lambda-\mu)(r-\mu)}.
\end{aligned}
$$

6.5 Comparative Statics

Table 1: Model Calibration

Model Calibration

Parameter	**Value**	**Source**
r	0.044	Average German Bund 10 year bond yield, 1996 - 2010.
c	16	Coupon rate 5%; national tax ratio 27.1% and debt-to-GDP ratio 87% based on OECD (2009) / IMF (2010).
λ	0.03	Author's assumption based on implied default intensities of 5Y CDS spreads of EMU banks, 2008 to 2010.
$\widetilde{\lambda} = \xi\lambda$	2λ	Author's assumption based on Borensztein & Panizza (2009).
k	0.18	Author's assumption based on Cecchetti et al. (2009).
Y_t	100	Normalized to 100.
π_d	0.68	Based on Moodys (2011) estimates, 1998 - 2008, $\pi_d = \pi_{j,1} = \pi_{j,2}$.
ϕ_d	0.03	Author's assumption based on Borensztein & Panizza (2009), $\phi_d = \phi_{j,1} = \phi_{j,2}$.
μ	0.03	Author's assumption.
σ	0.2	Author's assumption.

Table 2: Scenarios

Scenarios

Trade-openness	**Value**	**Description**
ϕ_d^{low}	0.02	Low trade-openness, $\phi_d^{low} = \phi_{j,1}^{low} = \phi_{j,2}^{low}$.
ϕ_d^{med}	0.03	Medium trade-openness, $\phi_d^{med} = \phi_{j,1}^{med} = \phi_{j,2}^{med}$.
ϕ_d^{high}	0.04	High trade-openness, $\phi_d^{high} = \phi_{j,1}^{high} = \phi_{j,2}^{high}$.

Financial sector	**Value**	**Description**
$k^{low}/\pi_{j,1}^{low}/\pi_{j,2}^{low}$	0.08/0.58/0.15	Small financial sector.
$k^{med}/\pi_{j,1}^{med}/\pi_{j,2}^{med}$	0.18/0.68/0.20	Medium financial sector.
$k^{high}/\pi_{j,1}^{high}/\pi_{j,2}^{high}$	0.28/0.78/0.25	Large financial sector.

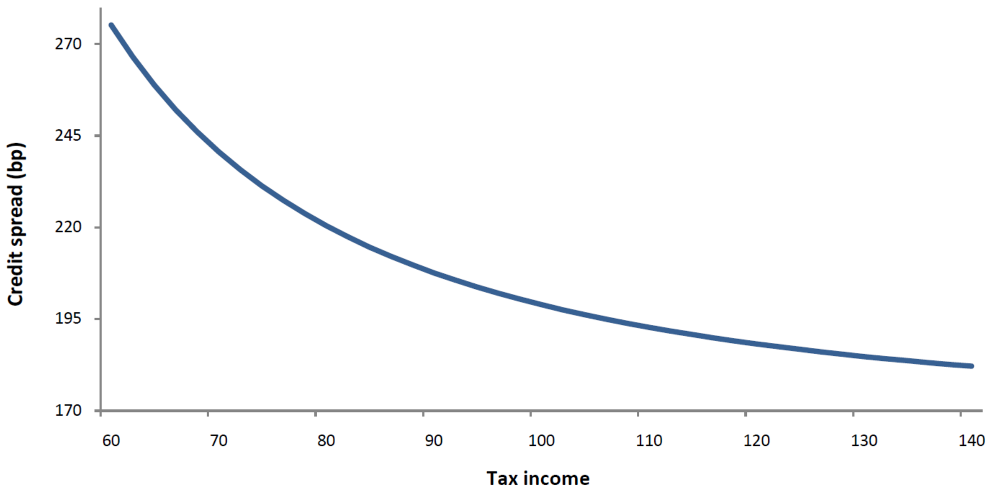

Figure 7: Credit spread vs. sovereign tax income Y

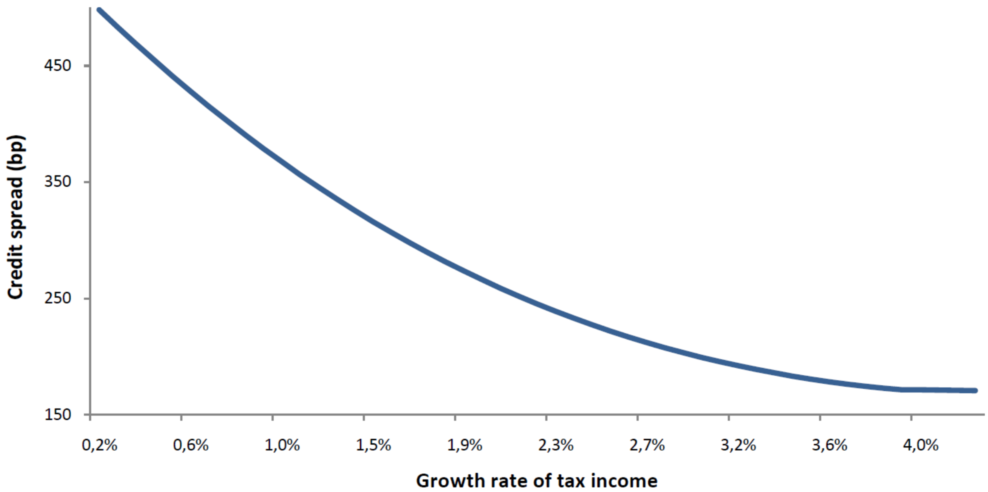

Figure 8: Credit spread vs. growth rate of tax income μ

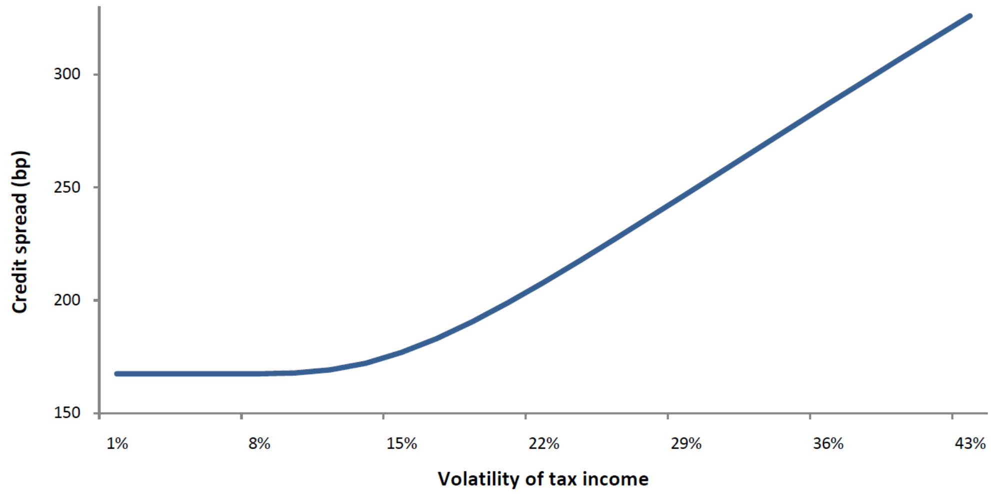

Figure 9: Credit spread vs. volatility of tax income σ

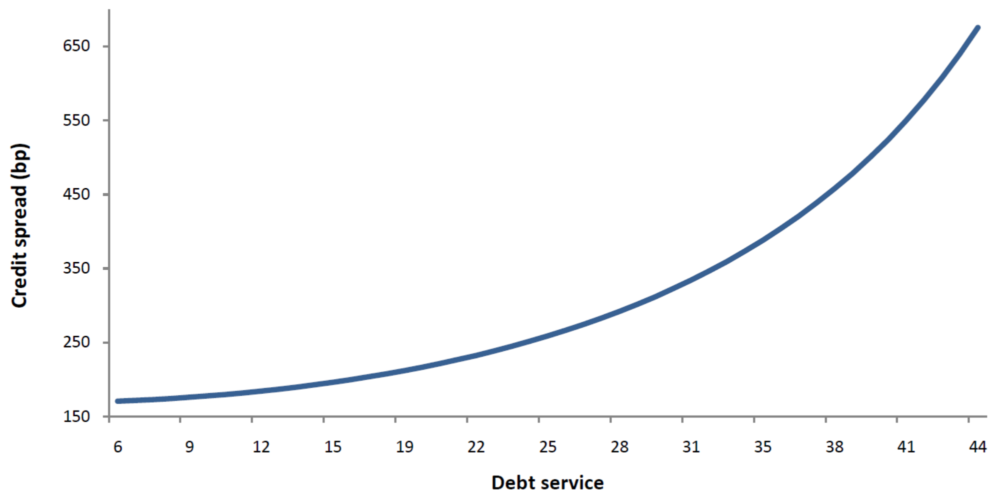

Figure 10: Credit spread vs. debt service c

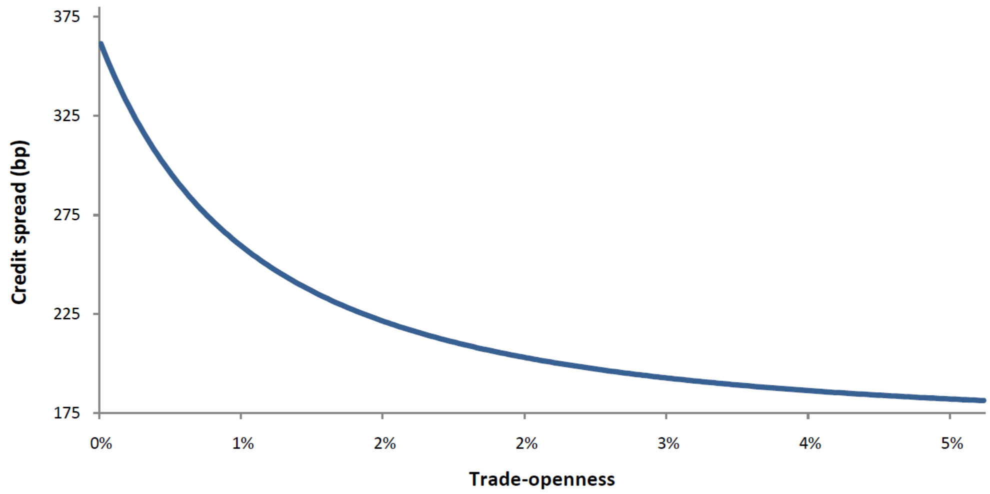

Figure 11: Credit spread vs. trade-openness ϕ

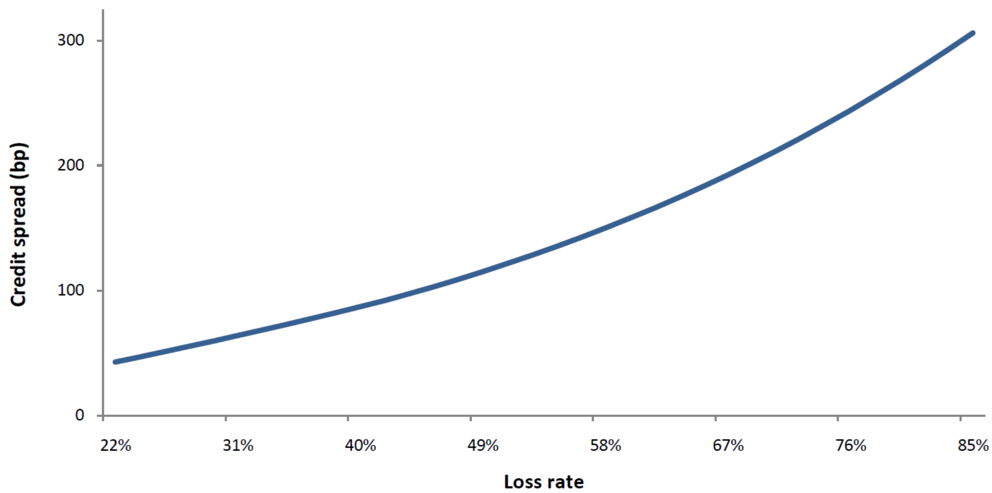

Figure 12: Credit spread vs. loss rate π

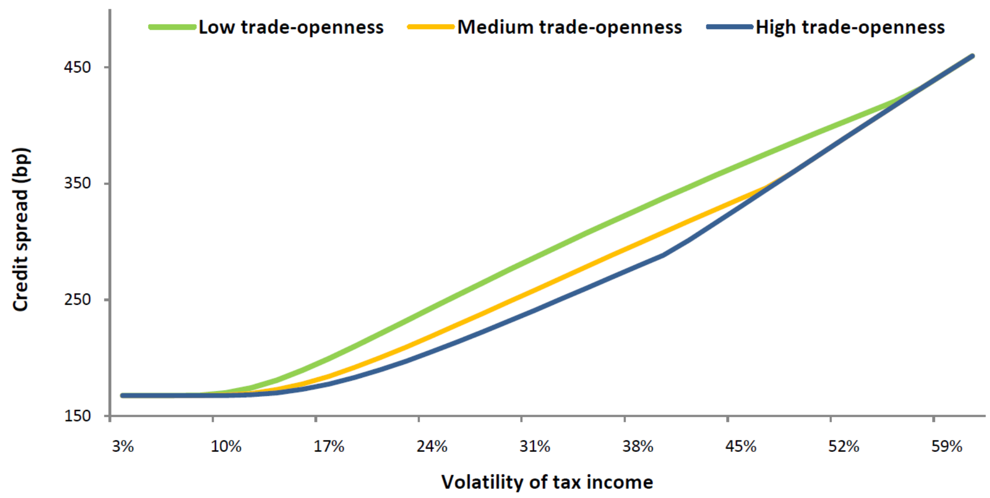

Figure 13: Credit spread vs. volatility of tax income σ

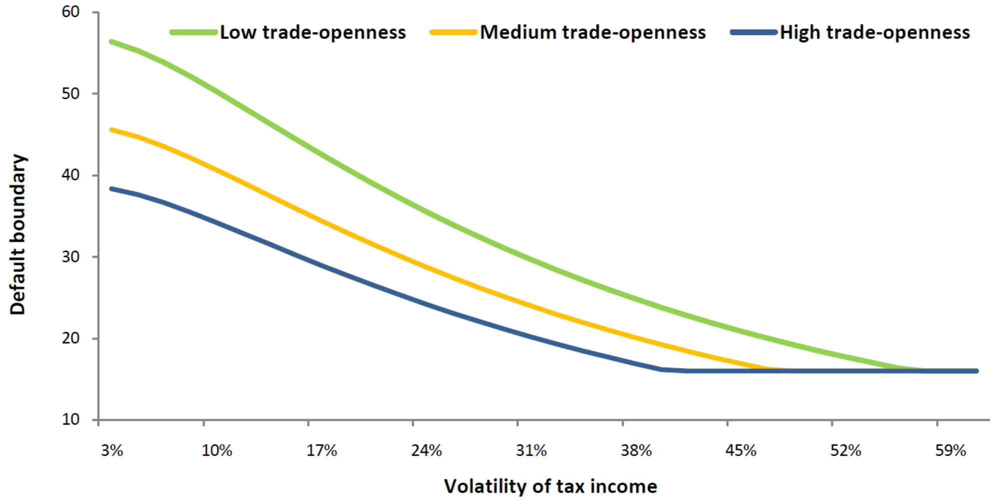

Figure 14: Default boundary vs. volatility of tax income σ

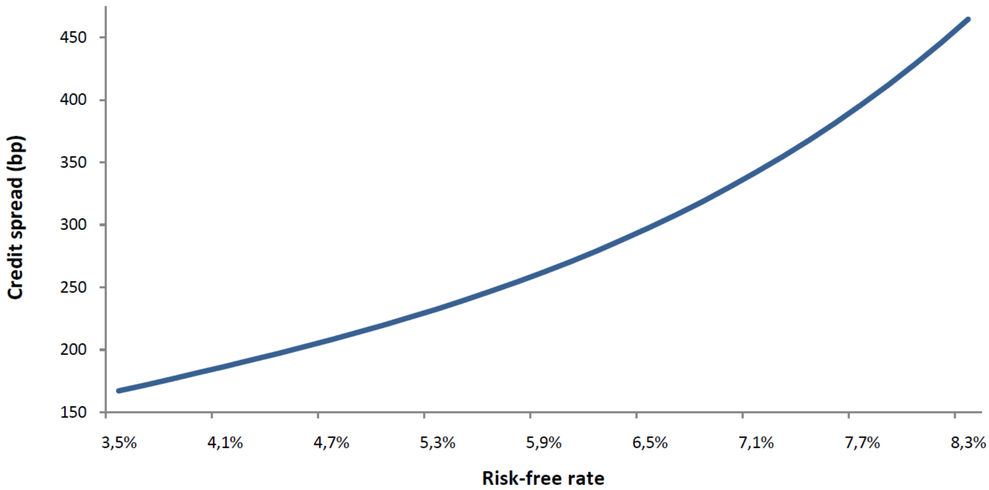

Figure 15: Credit spread vs. risk-free rate r

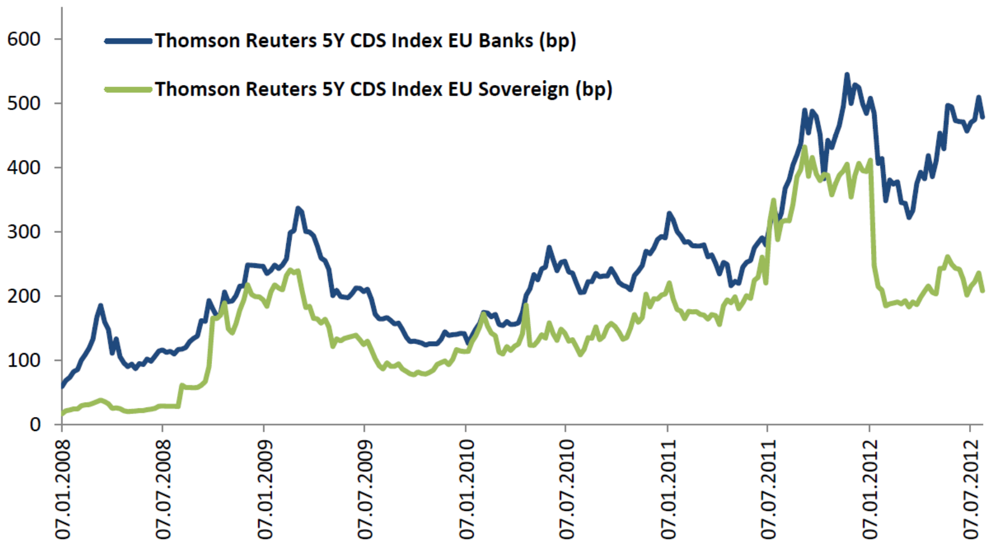

Figure 16: Thomson Reuters 5Y CDS Index EU Banks and Thomson Reuters 5Y CDS Index EU Sovereign

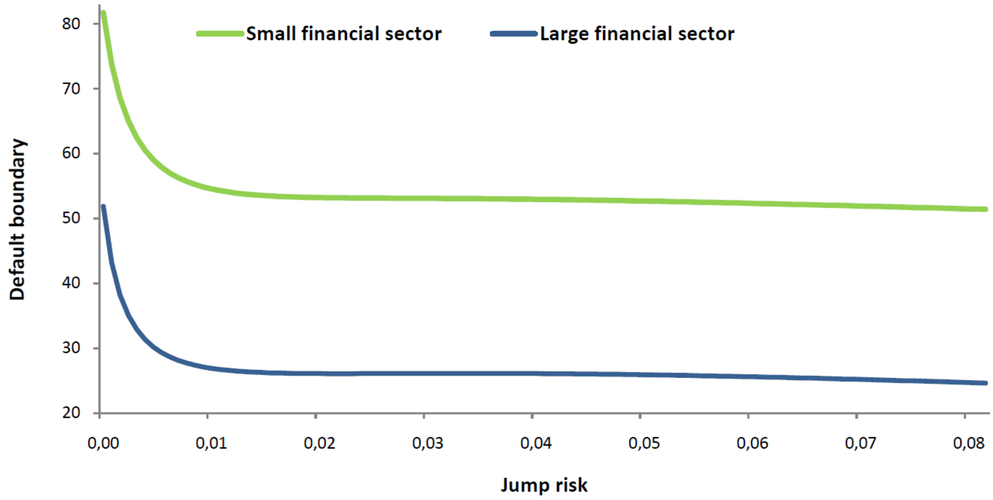

Figure 17: Default boundary vs. jump risk λ (low trade-openness)

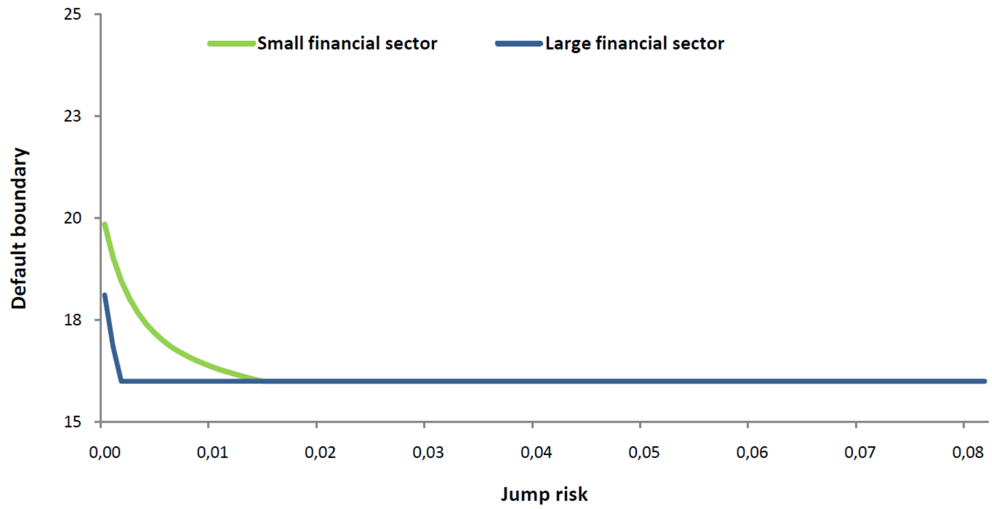

Figure 18: Default boundary vs. jump risk λ (high trade-openness)

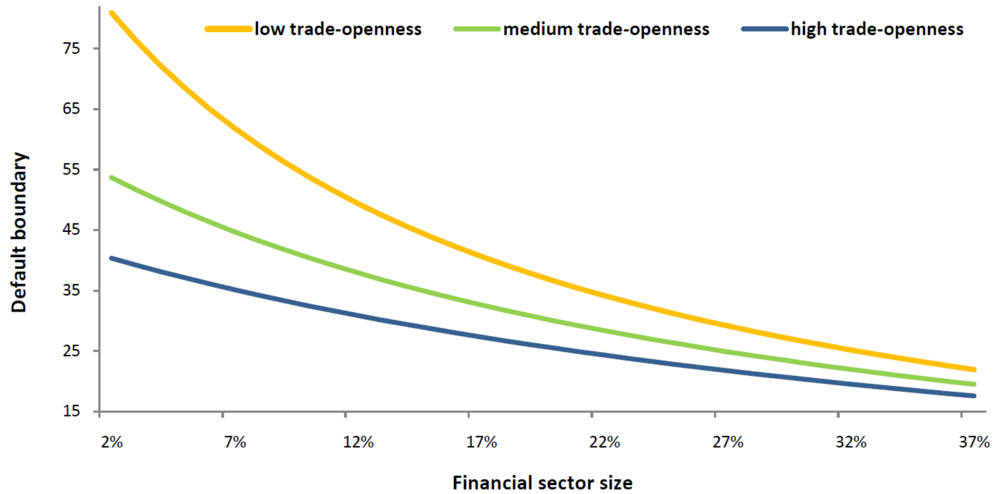

Figure 19: Default boundary vs. financial sector size k

In figures (4) and (19) I use the following parametrization to define the link between the jump size k and the loss rates $\pi_{j,1}$ and $\pi_{j,2}$:

$$\pi_{j,1}(k) \quad = \quad 0.50 + k \tag{75}$$

$$\pi_{j,2}(k) \quad = \quad 0.11 + 0.5k$$

Hence for $k^{low} = 0.08$, $k^{med} = 0.18$, and $k^{high} = 0.28$, loss rates amount to those reported in table (2): $\pi_{j,1}(k^{low}) = 0.58$, $\pi_{j,1}(k^{med}) = 0.68$, $\pi_{j,1}(k^{high}) = 0.78$, $\pi_{j,2}(k^{low}) = 0.15$, $\pi_{j,2}(k^{med}) = 0.2$, $\pi_{j,2}(k^{high}) = 0.25$.

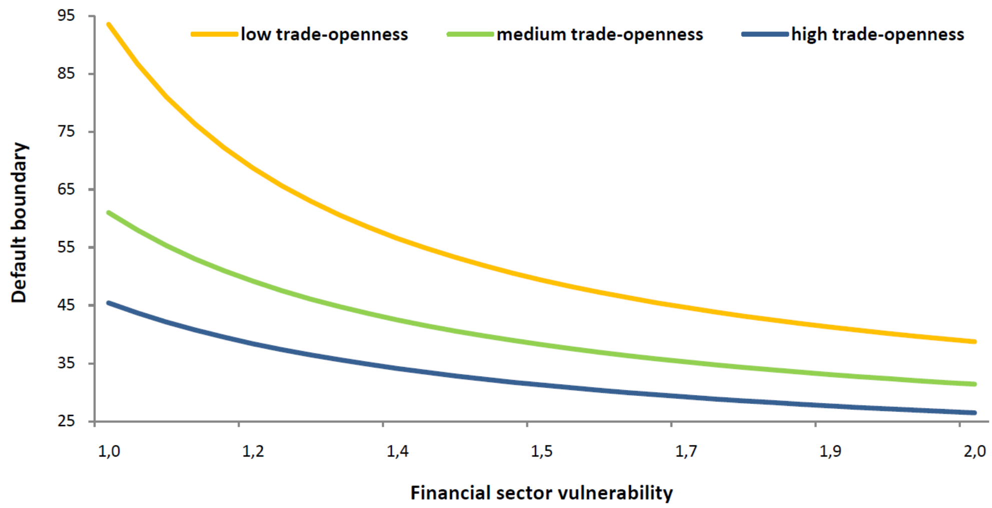

Figure 20: Default boundary vs. financial sector vulnerability ξ

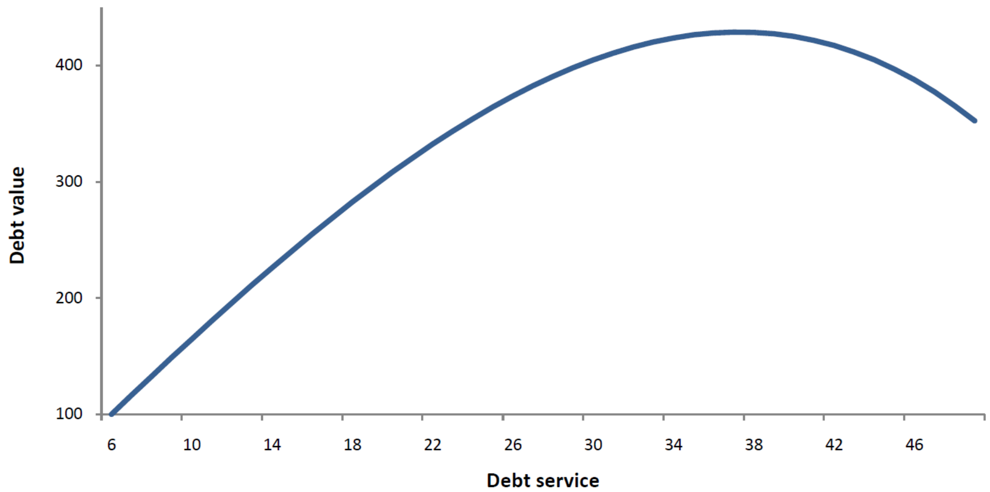

Figure 21: Debt Value vs. debt service c

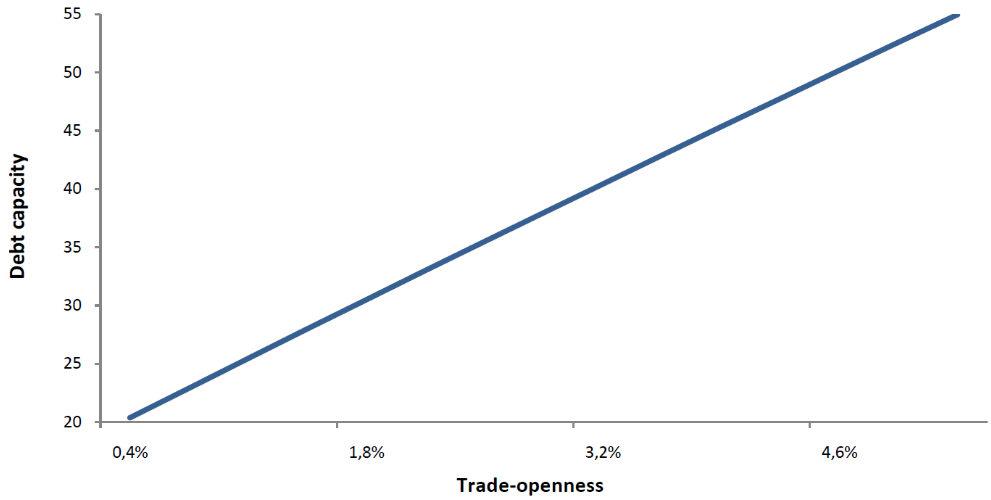

Figure 22: Debt capacity vs. trade-openness ϕ

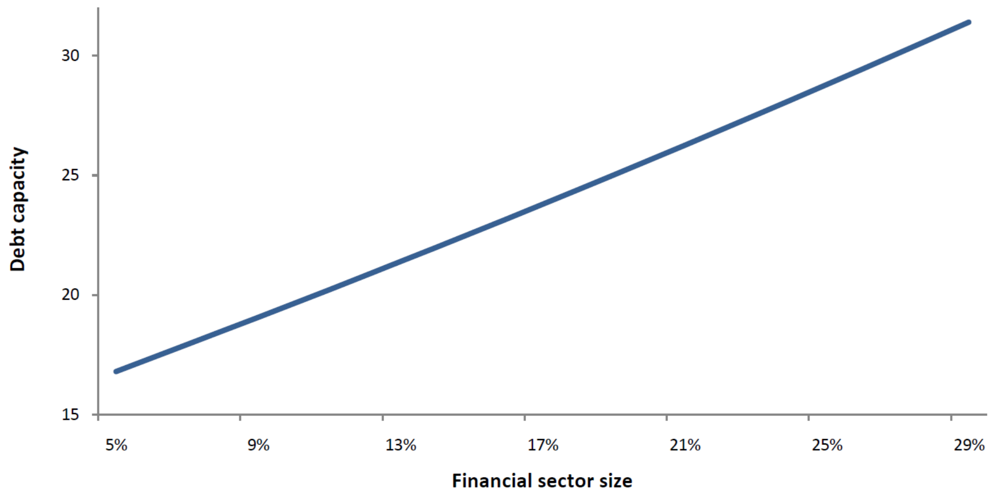

Figure 23: Debt capacity vs. financial sector size k

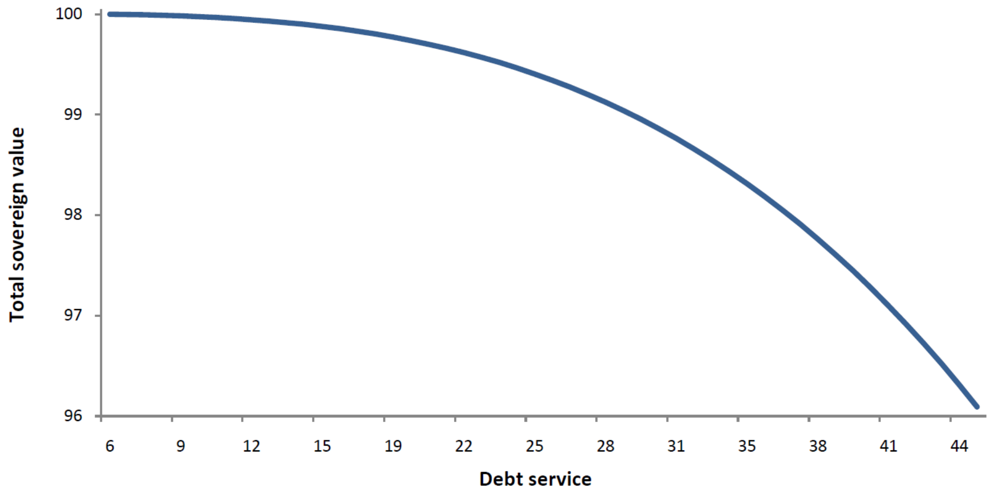

Figure 24: Total value of sovereign vs. debt service c

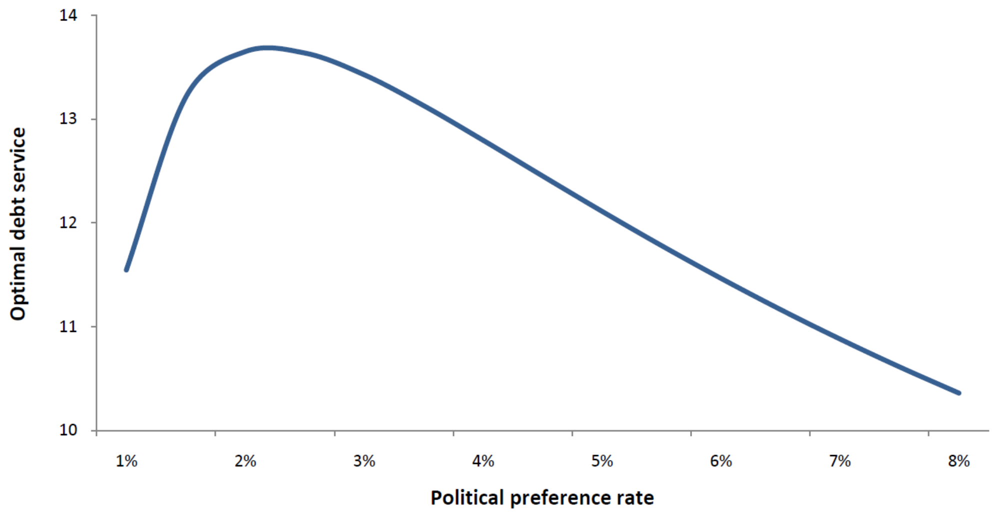

Figure 25: Optimal debt vs. political preference rate η

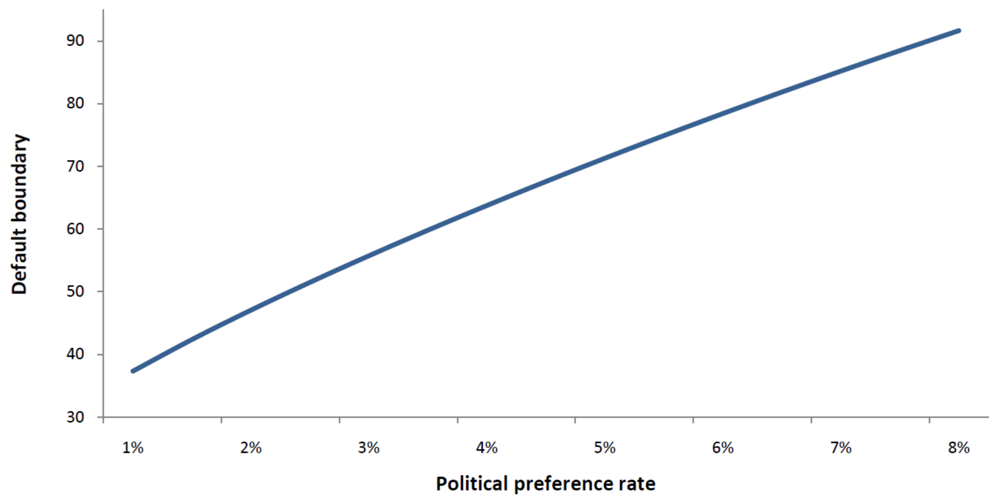

Figure 26: Default boundary vs. political preference rate η

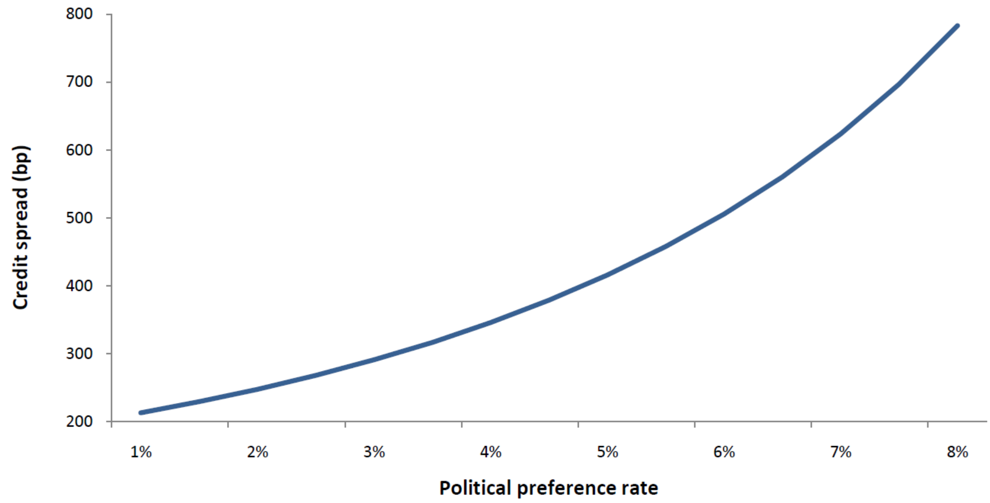

Figure 27: Credit spread vs. political preference rate η

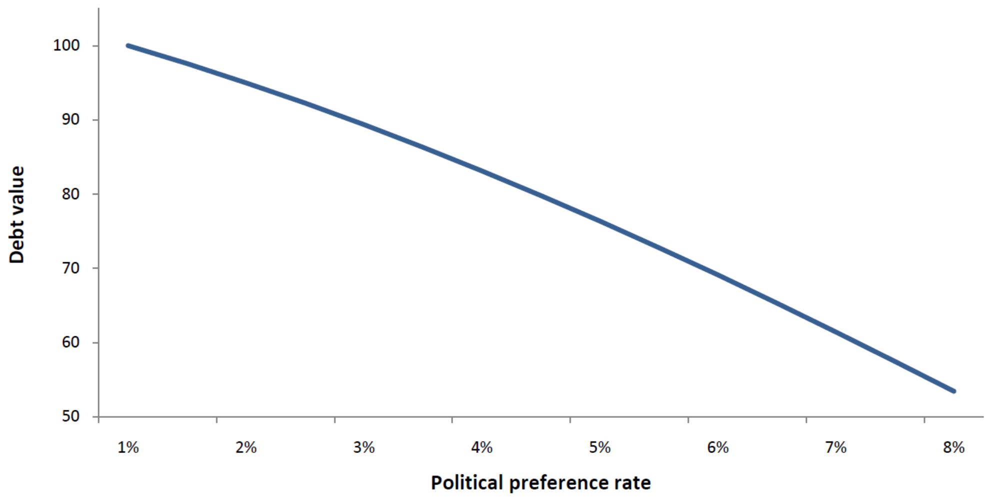

Figure 28: Debt value vs. political preference rate η

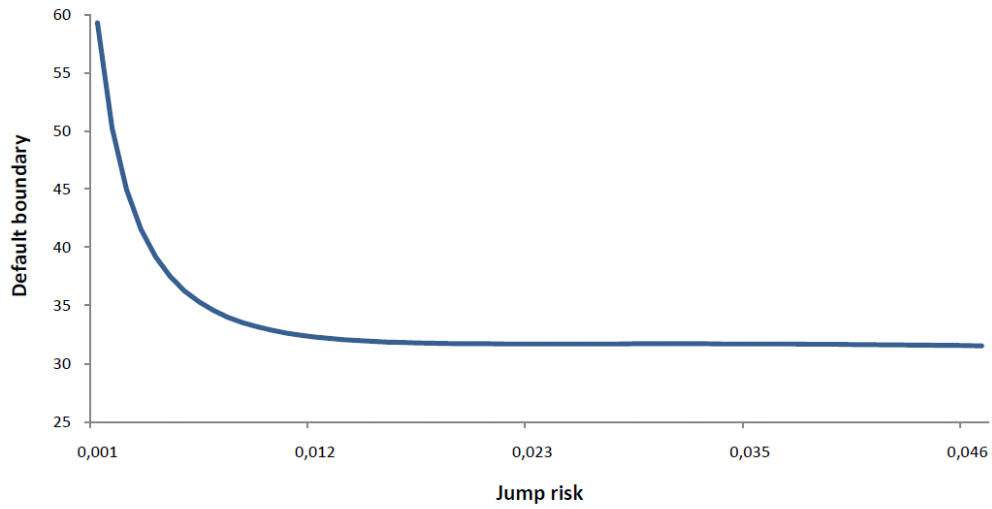

Figure 29: Default boundary vs. jump risk λ

6.6 Bank CDS Spreads

Table 3: Descriptive Statistics and Implied Default Intensities

CDS Spreads and Implied Default Intensities

	Period 1			Period 2		
Country	**Average**	**Min**	**Max**	**Average**	**Min**	**Max**
Austria	206.43	108.33	500.00	21.47	9.33	78.76
	(0.0342)	(0.0180)	(0.0831)	(0.0036)	(0.0015)	(0.0131)
Belgium	220.98	68.13	512.07	10.01	5,50	13.25
	(0.0367)	(0.0113)	(0.0851)	(0.0017)	(0.0009)	(0.0022)
Denmark	150.29	60.57	322.28	9.56	4.00	21.00
	(0.0249)	(0.0101)	(0.0535)	(0.0016)	(0.0007)	(0.0035)
France	93.09	57.18	179.30	9.67	5.67	14.50
	(0.0155)	(0.0095)	(0.0298)	(0.0016)	(0.0009)	(0.0024)
Germany	147.65	78.71	290.46	19.36	9.41	35.83
	(0.0245)	(0.0131)	(0.0483)	(0.0032)	(0.0016)	(0.0059)
Ireland	485.89	107.63	1858	10.46	5.75	15.35
	(0.0807)	(0.0179)	(0.3088)	(0.0017)	(0.0010)	(0.0025)
Italy	207.63	56.35	628.47	16.29	11.23	31.00
	(0.0345)	(0.0094)	(0.1043)	(0.0027)	(0.0019)	(0.0051)
Netherlands	144.42	51.36	310.91	12.32	4.80	20.73
	(0.0240)	(0.0085)	(0.0516)	(0.0020)	(0.0008)	(0.0034)
Portugal	486.50	68.20	1379	15.94	10.50	25.12
	(0.0808)	(0.0113)	(0.2291)	(0.0026)	(0.0017)	(0.0042)
Spain	416.27	149.38	937.02	15.94	10.50	25.12
	(0.0691)	(0.0248)	(0.1556)	(0.0026)	(0.0017)	(0.0042)
Sweden	119.81	66.42	241.25	16.54	10.12	24.69
	(0.0199)	(0.0110)	(0.0401)	(0.0027)	(0.0017)	(0.0041)
Average	**243.54**	**79.30**	**650.80**	**14.32**	**8.13**	**27.76**
	(0.0404)	**(0.0132)**	**(0.1081)**	**(0.0024)**	**(0.0013)**	**(0.0046)**
Median	**206.43**	**68.13**	**500.00**	**15.94**	**9.37**	**24.69**
	(0.0342)	**(0.0113)**	**(0.0831)**	**(0.0026)**	**(0.0015)**	**(0.0041)**

Table (3) presents descriptive statistics and implied default intensities of CDS spreads of European banks that were included in the 2010 EU-wide stress tests and where data was available on Datastream. Data frequency is weekly data and CDS spreads correspond to 5Y EUR-denominated contracts. Period 1 ranges from July 2008 to April 2012 while period 2 covers the time period between January 2004 and December 2006. Banks included are: Austria: Erste Bank, Raiffeisen Zentralbank; Belgium: KBC Group, Dexia; Denmark: Danske Bank; France: BNP Paribas, Credit Agricole, Societe Generale; Germany: Deutsche Bank, Commerzbank, Landesbank Baden-Württemberg, Bayrische Landesbank, DZ Bank, Deutsche Postbank, WestLB, HSH Nordbank, LB Hessen-Thüringen; Ireland: Bank of Ireland, Allied Irish Banks; Italy: UniCredit, Intesa Sanpaolo, UBI Banca; Netherlands: Fortis Bank, ING, Rabobank; Portugal: Caixa, BCP, BPI; Spain: CAM, Caja Madrid, Banco Popular Espanol, Banco Sabadell, Bankinter, Banco Pastor, Banco Bilbao;

Sweden: Nordea Bank, Skandinaviska Enskilda Banken, Svenska Handelsbanken, Swedbank. For Banco Pastor, Banco Bilbao, BPI, Rabobank, Bank of Ireland, and Dexia data is only available for period 1. For Allied Irish Banks the sample period ends in April 2011. For each country and each week the CDS spreads of the banks listed above are averaged. Then for each country the average value, the minimum value, and the maximum value over the sample period are calculated and reported. In each column the numbers in parentheses represent the corresponding implied default intensities that are estimated following standard industry practice as outlined in section (6.7).

6.7 CDS Valuation

The fair premium S_t^T of a CDS equates the premium and protection leg of the contract. The premium leg V_t^{prem} is the expected present value of premium payments made by the protection buyer to the protection seller until the contract matures or a credit event occurs:

$$V_t^{prem} = S_t^T RPV_t^T, \tag{76}$$

$$\begin{aligned} RPV_t^T &= \sum_{n=1}^{N} \delta(t_{n-1}, t_n) Z(t, t_n) Q(t, t_n) \\ &+ \sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} \delta(t_{n-1}, u) Z(t, u) Q(t, u)(-dQ(t, u)), \end{aligned} \tag{77}$$

where $t_0 = t$, $t_N = t + T$, N denotes the number of premium payments over the life of the CDS contract, and $\delta(t_{n-1}, t_n)$ refers to the day count fraction between two consecutive premium payment dates t_{n-1} and t_n. The variable $Z(t, u)$ denotes the price of a risk-free zero coupon bond at time t maturing at time u and $Q(t, u)$ refers to the risk-neutral survival probability until time u. Hence, the first term on the right hand side of equation (77) is the expected present value of premium payments conditional on surviving to the respective payments dates, while the second term captures the accrued premium to be paid if a credit event occurs between payment dates.

The protection leg V_t^{prot} is the expected present value of the protection payment made by the protection seller to the protection buyer if a credit event occurs:

$$V_t^{prot} = (1 - R) \int_t^{t+T} Z(t,u)(-dQ(t,u)), \tag{78}$$

where R denotes the recovery rate. Equating the premium and protection leg yields:

$$S_t^T = \frac{(1 - R) \int_t^{t+T} Z(t,u)(-dQ(t,u))}{RPV_t^T}. \tag{79}$$

The recovery rate R is set to the market convention of 40% and discount factors are computed based on the German zero yield curve. Modeling default as the first arrival time of a Poisson process with constant intensity λ, the risk-neutral probability of survival is given by: $Q(t,u) = \exp(-(u-t)\lambda)$. Based on equation (79) risk-neutral default intensities are estimated from observed CDS spreads.

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