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A discussion of the consistency axiom in cost-allocation problems*

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Abstract

The recent literature on cost allocation lacks consensus on what is an appropriate definition of the consistency axiom. We take this as evidence that a careful reexamination is necessary. The starting point of our critique is the widely adopted definition proposed in Moulin and Shenker (1994), which we show to be conceptually flawed. Rectifying this flaw leads to a definition of consistency which already appeared in the recent literature though without satisfactory conceptual justification. We offer a classification of the existing definitions of the consistency axiom by relating them to the definitions of consistency in cooperative games suggested in Davis and Maschler (1965) and Hart and Mas-Colell (1989). We argue that only the latter leads to a meaningful interpretation of consistency when production externalities are present.

1 Introduction

The *consistency axiom* has been widely studied in situations where a fixed resource must be allocated between a number of individuals. These situations include the allocation of a single private good in the presence (Young, 1987) or absence (Sönmez, 1994) of conflicting claims, the assignment of indivisible commodities (Sasaki, 95; Ehlers and Klaus, 2005), the allocation of several commodities in exchange economies (Tadenuma and Thomson, 1991; Thomson and Zhou, 1993), the problem of land division (Chambers, 2004), as well as matching (Sasaki and Toda, 1992; Toda, 2006) and bargaining problems (Lensberg, 1987; Thomson and Lensberg, 1989). We refer the reader to Thomson (2006) for a comprehensive survey of the literature on consistency in such allocation problems. The key principle behind the axiom is that of self-similarity: an allocation rule is consistent if the resource shares it (re-)allocates in any *reduced*

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problem, formed of a subset of the original population along with its allocation under this very rule, match the shares it allocates in the original problem (the *original shares*).

Consistency is an appealing axiom for at least two reasons. From a practical standpoint, only consistent allocation rules can actually be carried out. Indeed, because an inconsistent allocation rule could yield a reduced problem for which some agents receive (under the same rule) shares other than their original shares it is not clear how it should be implemented. Some individuals will prefer that it be applied to the original problem while others will claim that the reduced problem is the relevant level of application. Both arguments will be legitimate.

Consistency also bears a conceptual appeal. Since allocation rules are typically decided upon because they satisfy a number of desirable properties (e.g., fairness, efficiency, incentive compatibility), ensuring that these underlying principles are respected when considering subsets of individuals is essential. The consistency axiom does precisely that. In this regard, there exists an analogy between consistent allocation rules and self-similar mathematical objects, like fractals (see, e.g., Hutchinson, 1981), whose structure is preserved regardless of the level of zoom. In our framework, "zooming in" amounts to considering smaller and smaller subsets of individuals, along with their original allocation.

Some authors have also given a sequential interpretation of the consistency axiom (e.g., Lensberg, 1987; see also Thomson, 2006, and references therein): after a subset of agents has left the procedure with their allotted share, (re-)applying the same (consistent) allocation rule to the reduced problem yields the same outcome as if the allocation had been reached in a single blow. Therefore, one could potentially save on computational complexity by treating the original allocation problem as a succession of two-person allocation problems.

Although quite attractive, we shall show that this last interpretation is inappropriate when the resource to be allocated is not a fixed quantity, but a production process with possibly varying returns to scale. This fact has gone largely unnoticed in the literature on consistency in cost-allocation problems.[1] Consequently, the recent cost-sharing literature suffers from a lack of consensus as to how the consistency axiom should be interpreted, or even defined, thus resulting in several different definitions for the same axiom(see our literature review in Section 4). We take this as evidence that consistency is not a well-understood concept in the presence of production externalities. Our aim is to shed light on this understanding and to clarify the meaning of consistency in a production context.

2 Cost allocation

We consider the situation where a group of individuals jointly utilize a single production process. Each agent i demands a positive amount of consumption,

[1] To the best of our knowledge, only Moulin (2002) shows an understanding of why the sequential interpretation fails. However, Moulin and Shenker (1994) overlooks this issue entirely, as will be discussed later.

x_i, and the total cost of meeting the vector $x = (x_1, ..., x_n)$ of all demands, $C(x)$, must be exactly split between them. A *cost-sharing problem* is a triple, (N, C, x), where $N = \{1, ..., n\}$ is the set of individuals, $C : \mathbb{R}_+^N \to \mathbb{R}$ a cost function and $x \in \mathbb{R}_+^N$ a demand profile. We denote by Γ the set of all cost-sharing problems. A cost-sharing rule (or *sharing rule*) is a formula, $\varphi : \Gamma \to \mathbb{R}^N$, dividing the total cost as a function of the demands of the agents; i.e., such that the *budget is balanced*:

$$\sum_{i \in N} \varphi_i(N, C, x) = C(x).$$

We give three examples of prominent sharing rules of the literature on cost allocation.

The Equal-Split Rule (ES) shares costs equally between the agents, such that each pays $C(x)/n$.

Average Cost Pricing (ACP) shares costs in proportion of one's demand relative to total demand: agent i's cost share equals $\frac{x_i}{\sum_{j \in N} x_j} C(x)$.

The Serial Rule (SER), which has recently received much attention in the cost-sharing literature, is characterized by many desirable properties of fairness and incentive compatibility (see Moulin and Shenker, 1992, 1994). At its core is the idea that all individuals demanding a given level of output are equally responsible for the cost increment up to their joint demand level. On a three-person example where individuals are ordered such that $x_1 \leq x_2 \leq x_3$, and denoting $x^1 = (x_1, x_1, x_1)$ and $x^2 = (x_1, x_2, x_2)$, the serial cost shares can be written as follows:

$$\begin{cases} y_1 = & \frac{1}{3} C(x^1) \\ y_2 = & \frac{1}{3} C(x^1) + \frac{1}{2}[C(x^2) - C(x^1)] \\ y_3 = & \frac{1}{3} C(x^1) + \frac{1}{2}[C(x^2) - C(x^1)] + [C(x) - C(x^2)] \end{cases}$$

Naturally, ACP and SER make the most sense when individual demands are expressed in comparable units and, in particular, when the cost function is homogenous (i.e., when the total cost depends only on the total demand level).

These three rules share the common feature that they are defined by sensible normative principles: equality, proportionality and seriality. Whether or not one deems these principles compelling is beside the point of this work. We solely contend that each of these three rules is governed by a clear underlying logic and, hence, should pass any meaningful test of internal coherence, which is precisely what the consistency axiom should provide.

The starting point of our discussion will be Moulin and Shenker's (1994)—hereafter denoted MS94—interpretation of the consistency axiom as it is the most widespread and is representative of the common conceptual oversight pervading the literature which we intend to expose and rectify.

3 Defining the appropriate reduced problem

In order to define the consistency axiom, one must introduce the notion of a *reduced problem*, which is the (re-)allocation problem facing a subset of individuals, while ignoring the rest of the population.

We define the reduction of the cost-sharing problem (N, C, x) to coalition $S \subsetneq N$ to be a triple: (S, C^S, x_S), where $x_S \in \mathbb{R}_+^S$ is the restriction of x taking only the demands of the agents in S, and $C^S : \mathbb{R}_+^S \to \mathbb{R}$ is a *residual cost function* such that $C^S(x_S) = \sum_{i \in S} \varphi_i(N, C, x)$. Thus, the reduced problem can be thought of as a cost-sharing problem as well. The specification of how the residual cost function, C^S, is related to the original cost function, C, is central to our discussion. In the following definition of consistency, we take such a specification as given.

Definition 1 (Consistency) *A sharing rule, φ, is* consistent *if for any cost-sharing problem, (N, C, x), any subset $S \subsetneq N$, and any $i \in S$ the following holds:*

$$\varphi_i(S, C^S, x_S) = \varphi_i(N, C, x).$$

As mentioned in the introduction, several distinct definitions of the consistency axiom have been proposed in the cost-sharing literature. They differ in their interpretation of what is an appropriate residual cost function. The definition of a residual cost function given in MS94 is the following:[2]

$$C^S_{x_{-S}, y_{-S}}(z_S) = C(z_S, x_{-S}) - \sum_{i \in N \setminus S} y_i \qquad \text{for any } z_S \in \mathbb{R}_+^S, \tag{1}$$

where $y = \varphi(N, C, x)$ is the vector of original cost shares.

The reader may have noticed that we adapted the definition of MS94 to the heterogenous-goods case. Also, their definition involved a somewhat *ad hoc* truncation of the residual cost function by restricting attention to its non-negative part[3]. For the sake of exposition, we ignore this truncation operation as our criticism lies at a deeper, more conceptual level. Other (equally *ad hoc*) truncations have been suggested in the literature (see, e.g., Albizuri and Zarzuelo, 2005a and 2005b); our critique applies to those as well.

At the heart of expression (1) is the idea that once the cost share of the agents outside S has been determined, they can "put money on the table and depart without leaving an address: the remaining division problem can be conducted entirely without [them]." The image is from Moulin (2002) and is germane to the sequential interpretation of consistency mentioned in the introduction.

However, the MS94 definition of the consistency axiom fails our litmus test of the previous section: according to it, ES and ACP are consistent, but SER is not

[2] For notational brevity, we write x_{-S} and y_{-S} instead of $x_{N \setminus S}$ and $y_{N \setminus S}$.

[3] The same truncation also appears in Tijs and Koster (1998) and in Fleurbaey and Maniquet (1999)

(see MS94). Thus, either their definition, or our test, is inadequate, or both.[4] Either way, our observation calls for a thorough reexamination of Expression (1) as a definition of a residual cost. We argue that it suffers from a serious conceptual flaw:

On the one hand, requiring the reduced problem to be a cost-sharing problem implies the presence of a residual cost function. In turn, this implies that what the joint cost of the agents in S *could have been*—had they made demands other than the ones actually observed—matters. Hence the dependence of the residual cost on a vector of *hypothetical* demands, z_S, which may differ from the *actual* vector of demands, x_S.

On the other hand, considering the vector of cost shares for the agents outside of the coalition, y_{-S}, to be solely dependent on the actual demand vector implies that hypothetical demand profiles are irrelevant. Therein lies the flaw: if one wishes to compute the joint cost that agents in S *should have to pay under a given sharing rule* if they demanded z_S, one must acknowledge the fact that the shares of the agents outside S might depend on this hypothetical profile. In other words, Expression (1) should be modified in the following way:

$$C^S_{x_{-S},\varphi}(z_S) = C(z_S, x_{-S}) - \sum_{i \in S} \varphi_i(N, C, (z_S, x_{-S})) \qquad \text{for any } z_S \in \mathbb{R}^S_+, \tag{2}$$

where information on the sharing rule as well as on the cost function at hypothetical profiles is taken into account.

As it turns out, using expression (2) to construct residual cost functions yields a formal interpretation of the consistency axiom which passes our litmus test. I.e., ES, ACP and SER are consistent (see the Appendix). In fact, with this definition of the residual cost function, many sharing rules satisfy consistency. The reader can check that sharing costs according to fixed proportions, to path methods (see Friedman, 2004), dictatorial and priority rules, and two-part pricing (in the presence of fixed costs) are all consistent sharing rules. Thus, consistency is a very weak axiom, whose only role is to exclude "strange" rules, which is precisely what we are after. For instance, a rule allocating costs according to ES among N, but according to ACP among S when its cardinality is odd, and according to SER when it is even, is not consistent.

Note that a different approach altogether would be to view the production process as consisting of two stages: production, followed by distribution. The reduced problem would then amount to one of rationing (see Thomson, 2006, for a short discussion). We feel that such a decomposition might go against the very nature of the problem at hand when externalities are present (i.e., for most cost functions).

[4] As it turns out, both are inadequate: our test is, in fact, too weak, as it approves of Albizuri and Zarzuelo's (2005a, 2005b) definition of a reduced problem, which is guilty of the same conceptual flaw as that of MS94.

4 Relation to the literature and conclusion

The consistency axiom was originally introduced in the literature on cooperative games. The tension between two different definitions of what is a reduced game exists there as well. In Davis and Maschler (1965), the reduced game for a coalition considers what remains after the agents outside the coalition take their share of the grand coalition surplus (i.e. at the *actual* profile). Expression (1) is clearly the cost-sharing version of that interpretation.

By contrast, Hart and Mas-Colell (in Hart and Mas-Colell, 1989, hereafter HMC) chose to acknowledge the dependence of the shares of the "departing" individuals on the hypothetical participation of the "remaining" agents when defining the reduced game for a coalition. Expression (2) is the cost-sharing analog of their definition.

Nevertheless, while their views differ on what constitutes a reduced game, these authors stress that the definition adopted should be relevant to the particular question at hand (See HMC, and Maschler, 1990). In light of the preceding discussion, our interpretation of their warning is that the Davis-Maschler definition may be appropriate for allocating a fixed resource but clearly is not when production externalities are present; the HMC definition is the appropriate one in that case.

Except for the question of allocating costs, most allocation problems encountered in the literature on consistent allocation dealt with distributing a fixed resource (see Thomson 2006). The consensus, in this case, is to adopt the Davis-Maschler definition of consistency. This may help explain the fact that the early literature on consistency in cost sharing (MS94) also used the Davis-Maschler definition, perhaps out of habit.

Since then, attempts to properly define the consistency axiom in cost-sharing problems can be sorted into two categories: those akin to the Davis-Maschler definition (e.g., MS94; Tijs and Koster, 1998; Fleurbaey and Maniquet, 1999; Albizuri and Zarzuelo, 2005a and 2005b; Koster, 2006) and those related to the HMC definition (Tijs and Koster, 1998[5]; Friedman, 2004; McLean, Pazgal and Sharkey, 2004)

Maurice Koster (see Koster, 2006) recently developed an intriguing axiom related to consistency in the case where the cost function is homogenous. He takes the view that the resource to be shared is the cost function itself and defines the residual cost function of a coalition to be the original one minus the portion of the original cost function virtually allocated to the "departing" agents at the original demand profile (it is, therefore, a definition of the Davis-Maschler type). This definition is problematic for at least two reasons. First, it seems to incorporate a notion of responsibility of sorts, in the sense that it aims to hold agents responsible for the increase in costs due to their presence,

[5] Tijs and Koster (1998) considers two different definitions of consistency, one of each type. While the authors seem to favor the definition of the HMC type, they do not provide a satisfactory justification for their inclination. Moreover, given that subsequent work by Koster (see Koster, 2006) reverts to a Davis-Maschler approach to consistency, one may wonder whether the conceptual distinction between the two approaches had been clearly identified.

which is not quite in line with the spirit of consistency. But most importantly, it fails to recognize a sharing rule as straightforward as ES as consistent when externalities are present.

We believe the definition in Friedman (2004)—which exactly corresponds to Expression (2)—to be most faithful to the notion of consistency[6] and the results therein to be among the most meaningful contributions to the topic of consistency in cost sharing. Nonetheless, even Friedman failed to recognize the large conceptual gap between his definition (of the HMC type) and that of MS94 (of the Davis-Maschler type) when justifying the use of his definition:

> "[It] is a natural extension of the version used in [HMC] for TU games and by [MS94] for cost sharing problems in which the cost function is required to be homogenous."[7]

This serious oversight serves as further evidence that the present discussion was necessary.

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[6] McLean et al. (2004) uses the same definition. However, their framework is less general because they focus on setting a constant per-unit price for each agent.

[7] Abbreviations were added for the sake of "consistency" with the rest of the paper.

5 Appendix: Proof that ES, ACP and SER are consistent in the HMC sense

Consider a cost-sharing problem $(N, C, x) \in \Gamma$ as well as a coalition $S \subset N$ of agents. Denote by $L = N \backslash S$ the set of agents who "leave" the procedure and by $(S, C^S_{x_L,\varphi}, x_S)$ the reduced problem of the agents in S (i.e., the agents who "stay"), with $C^S_{x_L,\varphi}$ defined as in Expression (2). We shall show that $\varphi_i(S, C^S_{x_L,\varphi}, x_S) = \varphi_i(N, C, x)$ for all $i \in S$ when φ is ES, ACP and SER, respectively.

5.1 Proof that ES is consistent

In this section, we let $\varphi \equiv ES$. It is immediate from Expression (2) that the residual cost function for coalition S equals the following:

$$C^S_{x_L,\varphi}(z_S) = \frac{n - |S|}{n} C(N, C, (z_S, x_L)).$$

Clearly, applying ES to the reduced problem, amounts to dividing this residual cost by $n - |S|$, which is equivalent to splitting the original cost equally between the n agents.

5.2 Proof that ACP is consistent

In this section, we let $\varphi \equiv ACP$. It follows from Expression (2) that the residual cost function for coalition S can be written as:

$$C^S_{x_L,\varphi}(z_S) = \frac{\sum_{i \in S} z_i}{\sum_{j \in L} x_j + \sum_{i \in S} z_i} C(N, C, (z_S, x_L)).$$

This residual cost, when split proportionally to the size of the z_i's yields exactly the original ACP shares when $z_S = x_S$.

5.3 Proof that SER is consistent

The consistency of SER is a corollary of a more general result in Friedman (2004) but we provide a proof nonetheless, for the sake of self-containedness. Also, because our proof is specific to SER, it may provide the reader with a better intuition for the result.

In this section, let $\varphi \equiv SER$. We first introduce some notation. Let $S = (s_1, ..., s_{|S|})$, and for any integer $j \in \{1, ..., |S|\}$, we denote by $x^j_S = (x_{s_1}, ..., x_{s_{j-1}}, x_{s_j}, x_{s_j}, ..., x_{s_j})$, the vector of size $|S|$ obtained by replacing the last $|S| - j + 1$ coordinates of x_S with x_{s_j}. Without loss of generality, we shall assume that the agents in N are ordered in increasing order of their demand levels: $x_1 \leq x_2 \leq ... \leq x_n$ and that the same holds for agents in S ($x_{s_j} \leq x_{s_{j+1}}$ for all j). To economize on notation, we shall often denote by $y_i = \varphi_i(N, C, x)$ agent i's cost share in the original problem, and by $y'_i = \varphi_i(S, C^S_{x_L,\varphi}, x_S)$ her share in the reduced problem of coalition S. We also abuse notation slightly and write $\varphi_S(N,C,x) = \sum_{i\in S}\varphi_i(N,C,x)$ and $\varphi_L(N,C,x) = \sum_{i\in L}\varphi_i(N,C,x)$.

Next, we turn to two characteristic properties of SER:

- Equal treatment of equals (ETE): for any $j,k \in S$, $x_j = x_k \implies [y_j = y_k$ and $y'_j = y'_k]$,
- Independence of higher demands (IHD): for any $j \in S$, $\varphi_{s_j}(N,C,(x^j_S,x_L)) = \varphi_{s_j}(N,C,(x_S,x_L))$.

We proceed by induction. We first consider agent s_1's cost share:

$$\begin{aligned}
y'_{s_1} &= \frac{1}{|S|}C^S_{x_L,\varphi}(x^1_S) \\
&= \frac{1}{|S|}\left(C(x^1_s,x_L) - \varphi_L(N,C,(x^1_S,x_L))\right) \\
&= \frac{1}{|S|}\varphi_S(N,C,(x^1_S,x_L)) \qquad \text{by budget balance} \\
&= \frac{1}{|S|}|S|\varphi_{s_1}(N,C,(x^1_S,x_L)) \qquad \text{by ETE} \\
&= \varphi_{s_1}(N,C,x) \qquad \text{by IHD} \\
&= y_{s_1}.
\end{aligned}$$

Now, fix $i \in S$, and suppose we established that $y'_{s_j} = y_{s_j}$ for all $j \le i$. By the definition of SER,

$$y'_{s_i} = y'_{s_{i-1}} + \frac{1}{|S|+1-i}\left(C^S_{x_L,\varphi}(x^i_S) - C^S_{x_L,\varphi}(x^{i-1}_S)\right)$$

where, by budget balance, ETE and IHD, and the induction hypothesis, respectively, we get $C^S_{x_L,\varphi}(x^{i-1}_S) = \varphi_S(N,C,(x^{i-1}_S,x_L)) = \sum_{k=1}^{i-2} y'_{s_k} + (|S|-i+2)y'_{s_{i-1}} = \sum_{k=1}^{i-2} y_{s_k} + (|S|-i+2)y_{s_{i-1}}$. Similarly, $C^S_{x_L,\varphi}(x^i_S) = \varphi_S(N,C,(x^i_S,x_L)) = \sum_{k=1}^{i-1} y'_{s_k} + (|S|-i+1)y_i$. It follows immediately that $y'_{s_i} = y_{s_i}$. Therefore, SER is consistent in the HMC sense.

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