FRACTIONS OPERATIONS

Summary

1. Elements of a fraction

The fraction $\frac{\boldsymbol{a}}{\boldsymbol{b}}$ is composed of a numerator $(\boldsymbol{a})$ and a denominator $(\boldsymbol{b})$.

2. Equivalent fractions

It is important to remember that there are many ways to represent the same fraction. For example, the fractions $\frac{\mathbf{1}}{\mathbf{2}}$ and $\frac{\mathbf{2}}{\mathbf{4}}$ are equivalent. But how do we go from one fraction to another and conserve the equivalence relation?

A fraction remains equivalent if the numerator *and* the denominator are multiplied or divided by the same number.

Example

$$\frac{2}{3}=\frac{2\times 5}{3\times 5}=\frac{10}{15}$$

$$\frac{24}{30}=\frac{24\div 6}{30\div 6}=\frac{4}{5}$$

3. Simplification of a fraction

A fraction is written in its simplified form if the numerator and the denominator have no common factor. In other words, it is impossible to find a number that is a divisor to both the numerator and the denominator in a fraction's simplified form.

Example

The fraction $\frac{120}{200}$ is not written in its simplified form since there are numbers that divide both 120 and 200. The largest common divisor (factor) of 120 and 200 is 40, where

$$\frac{120}{200} = \frac{120 \div 40}{200 \div 40} = \frac{3}{5}$$

Since we divided the numerator and the denominator by the same number (40), the fraction $\frac{3}{5}$ is equivalent to $\frac{120}{200}$. In addition, $\frac{3}{5}$ is the simplified form of $\frac{120}{200}$ since no other common factor exists for 3 and 5.

A simplification can be done in many steps if we do not recognize at once the largest common factor for the numerator and the denominator.

Example

$$\frac{108}{144} = \frac{108 \div 2}{144 \div 2} = \frac{54}{72} = \frac{54 \div 9}{72 \div 9} = \frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

"Countdown" enthusiasts will have noticed that 108 and 144 have 36 as a common factor:

$$\frac{108}{144} = \frac{108 \div 36}{144 \div 36} = \frac{3}{4}$$

In the end, no matter how many steps taken, the same simplified form will be found...

4. Rules for adding and subtracting fractions

$$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$$

The symbol $\pm$, which is read "plus or minus", indicates that this rule applies both to sums and subtractions.

Example

$$\frac{3}{8} + \frac{7}{8} = \frac{3+7}{8} = \frac{10}{8} = \frac{5}{4}$$

$$\frac{5}{6} + \frac{7}{6} = \frac{5-7}{6} = -\frac{2}{6} = -\frac{1}{3}$$

Note that the adding and subtracting rule for fractions is applicable only if both fractions have the same denominator. However, this will generally not be the case. We will need to rewrite the fractions into equivalent fractions with a common denominator.

Example

Evaluate the following sum $\frac{2}{5} + \frac{1}{3}$

These fractions cannot be added together before rewriting them with a common denominator. The smallest common multiple of 3 and 5 is 15. 15 will therefore be the common denominator.

$$\frac{2}{5} + \frac{1}{3} = \frac{2 \times 3}{5 \times 3} + \frac{1 \times 5}{3 \times 5} = \frac{6}{15} + \frac{5}{15} = \frac{6+5}{15} = \frac{11}{15}$$

Example

Evaluate the following subtraction $\frac{3}{8} - \frac{17}{24}$

The common denominator (the smallest common multiple) of 8 and 24 is 24. The fraction $\frac{17}{24}$ does not require rewriting. However, $\frac{3}{8}$ must be written so that 24 is its denominator.

$$\frac{3}{8} - \frac{17}{24} = \frac{3 \times 3}{8 \times 3} - \frac{17}{24} = \frac{9}{24} - \frac{17}{24} = \frac{-8}{24} = \frac{-1}{3}$$

Note : When possible, it may be useful to simplify fractions before proceeding with addition or subtraction. Such a simplification will facilitate finding a common denominator.

Example

Evaluate the following sum $\frac{9}{12} + \frac{7}{14}$

$$\frac{9}{12} + \frac{7}{14} = \frac{9 \div 3}{12 \div 3} + \frac{7 \div 7}{14 \div 7} = \frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4} = \frac{5}{4}$$

In the previous example, the common denominator of 12 and 24 would have been 84. By first simplifying each fraction, the calculations were greatly reduced.

5. Multiplication rule for two fractions

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

It is important to note that contrary to sums, the multiplication rule does not impose constraints to the denominator values. This means they do not need to be common.

Example

$$\frac{4}{7} \times \frac{3}{11} = \frac{4 \times 3}{7 \times 11} = \frac{12}{77}$$

$$\frac{-3}{2} \times \frac{5}{4} = \frac{-3 \times 5}{2 \times 4} = \frac{-15}{8}$$

Note : It may be useful to simplify fractions before multiplying. In addition to simplifying each fraction individually, simplifying the denominator of one fraction with the numerator of the other fraction is permitted, *provided that both have common factors.*

Example

Evaluate the following product $\frac{27}{16} \times \frac{8}{81}$.

You will note that the two fractions, $\frac{27}{16}$ and $\frac{8}{81}$ are already simplified. Nonetheless, the denominator 16 and the numerator 8 have 8 as a common factor. The denominator 81 and the numerator 27 have 27 as common factor. It is therefore possible to simplify before multiplying

$$\frac{27}{16} \times \frac{8}{81} = \frac{27 \div 27}{16 \div 8} \times \frac{8 \div 8}{81 \div 27} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

6. Division rule of two fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

The rule allows us to transform a division into a multiplication.

Example

$$\frac{2}{7} \div \frac{3}{8} = \frac{2}{7} \times \frac{8}{3} = \frac{16}{21}$$

Pay attention to the following notation that describes the same division :

$$\frac{2/7}{3/8} = \frac{2}{7} \times \frac{8}{3} = \frac{16}{21}$$

A few final remarks

- Working with fractions does not modify the priority of operations.

Example

$$\frac{2}{3} + \frac{4}{5} \times \frac{2}{3} = \frac{2}{3} + \frac{8}{15} = \frac{10}{15} + \frac{8}{15} = \frac{18}{15} = \frac{6}{5}$$

- A whole number can always be written as a fraction if an operation is to be done between the number and a fraction.

Example

$$8 - \frac{5}{3} = \frac{8}{1} - \frac{5}{3} = \frac{24}{3} - \frac{5}{3} = \frac{19}{3}$$

- Avoid working with mixed numbers... transform them into simple fractions.

Example

$$4^{2}/_{7} = 4 + \frac{2}{7} = \frac{28}{7} + \frac{2}{7} = \frac{30}{7}$$

7. Exercises - Operations with numbers

Calculate the following while respecting the priorities :

a) $8 - 2 \times (3 - 2 \times 5)$

b) $4 - 2 \times (-4) + 6 \div 2$

c) $8 - 2 \times 5 + 2 \times ((-2) - 3)$

d) $(8 - 2) \times (-3) - 2 \times ((-1) - 6)$

e) $\frac{2}{7} - \frac{5}{3} \times \frac{27}{35}$

f) $\frac{5}{8} + \frac{9}{25} \div \frac{18}{15}$

g) $\frac{2}{7} - \frac{5}{3}$

h) $\frac{1}{4} + \frac{3}{5}$

i) $\frac{5}{36} \times \frac{9}{25}$

j) $\frac{3}{8} \div \frac{4}{5}$

Solutions

a) 22

b) 15

c) -12

d) -4

e) -1

f) $37/40$

g) $-29/21$

h) $17/20$

i) $1/20$

j) $15/32$