INTEREST RATE CONVERSION

Summary

1. Vocabulary

- Interest dates: dates when interest is received;
- Interest period: time interval between two interest dates;
- Periodic interest rate: real interest rate per interest period;
- Capitalization: adding interest to the capital;
- Nominal interest rate: This rate, calculated on an annual basis, is used to determine the periodic interest rate. Generally, this is the rate that is published. It should always be accompanied by the type of capitalization. For example, a rate of "8 % capitalized biannually" means that the interest period is half-yearly, and the periodic interest rate (biannual) is $\frac{8\,\%}{2} = 4\,\%$. The nominal interest rate does not correspond to the effective annual interest rate, unless the capitalization is annual;
- Effective interest rate: effective annual interest rate.

2. Equivalence of interest rates

Imagine the following situation: a bank offers you an effective annual interest of 6 %; a bank B offers you a periodic interest rate of 1,5 % per quarter. How would you determine which bank offers the best yield? To compare two interest rates, you need to be able to evaluate them during the same period. For example, we can find the annual interest rate equivalent to a quarterly interest rate of 1,5 % and verify if it is greater than 6 %. This **conversion** must be done respecting the value of an investment that would be accumulated at a given interest rate.

Definition

> Two rates are said to be **equivalent** if, for the same initial investment and over the same time interval (one full year, for example), the final value of the investment, calculated with the two interest rates, is equal.

Consider a case where we deposit an amount V0 in a bank offering a quarterly interest rate i_{quart}. At the end of the year, i.e. 4 quarters, the value of the investment will be:

$$V_0(1 + i_{quart})^4$$

Now, suppose that another bank offers an annual interest rate i_{ann}. At the end of one full year, the value of the investment will be:

$$V_0(1 + i_{ann})^1.$$

According to the definition, the rates i_{ann} and i_{quart} are equivalent if

$$V_0(1 + i_{quarttrim})^4 = V_0(1 + i_{ann})^1$$

$$\Leftrightarrow (1 + i_{quart})^4 = (1 + i_{ann})^1$$

Note that the initial investment value is of no importance. This relation allows us to pass from a quarterly interest rate to an equivalent annual interest rate or vice versa.

Example 1

A bank A offers you an (effective) annual interest rate of 6 %; the bank B offers an interest rate of 1,5 % per quarter. Which of these two banks offers the best return?

Solution

Bank B offers a quarterly rate of 1,5 %. The equivalent annual interest rate (or effective rate) for this interest rate can be obtained by the relation

$$(1 + \mathrm{i}_{\mathrm{quart}})^4 = 1 + \mathrm{i}_{\mathrm{ann}}$$
$$(1{,}015)^4 = 1 + \mathrm{i}_{\mathrm{ann}}$$
$$\mathrm{i}_{\mathrm{ann}} = (1{,}015)^4 - 1 = 0{,}06136355$$

Bank B, therefore offers a better return (with (effective) annual interest rate of 6,136355 %) *than bank A.*

Alternative solution

Bank A offers an effective interest rate of 6 %. The quarterly equivalent is also obtained from the relation :

$$(1 + \mathrm{i}_{\mathrm{quart}})^4 = 1 + \mathrm{i}_{\mathrm{ann}}$$
$$(1 + \mathrm{i}_{\mathrm{quart}})^4 = 1 + 0{,}06 = 1{,}06$$
$$\left((1 + \mathrm{i}_{\mathrm{quart}})^4\right)^{1/4} = (1{,}06)^{1/4}$$
$$1 + \mathrm{i}_{\mathrm{quart}} = (1{,}06)^{1/4}$$
$$\mathrm{i}_{\mathrm{trim}} = (1{,}06)^{1/4} - 1 = 0{,}014673846 \text{ i.e } 1{,}4673846\ \%$$

This rate is inferior to the quarterly interest rate offered by bank B and we arrive at the same conclusion.

The reasoning we just made applies to all interest rate conversions. A periodic interest rate can always be converted given that the rate equivalence relation is respected :

Interest rate equivalence relation

$$(1+i_{ann})^1 = (1+i_{bian})^2 = (1+i_{quart})^4 = (1+i_{mon})^{12}$$

We must make sure this relation is satisfied so that the accumulated value of a 1$ investment, capital and interest, is the same at the end of one full year no matter which method of capitalization is used.

Example 2

What is the monthly equivalent interest rate to a quarterly interest rate of 2,5 %?

Solution

We want to find i_{mon} knowing that $i_{quart} = 2{,}5$ %. According to the relation of rate equivalence, the equality

$$(1+i_{quart})^4 = (1+i_{mon})^{12}$$

must be satisfied. i_{mon} needs to be isolated:

$$(1+i_{mon})^{12} = (1+i_{quart})^4$$
$$(1+i_{mon})^{12} = (1+2{,}5\%)^4$$
$$((1+i_{mon})^{12})^{1/12} = ((1{,}025)^4)^{1/12}$$
$$(1+i_{mon})^{12/12} = (1{,}025)^{4/12}$$
$$1+i_{mon} = (1{,}025)^{1/3}$$
$$i_{mon} = (1{,}025)^{1/3} - 1$$
$$i_{mon} = 0{,}8265\%$$

Example 3

What is the monthly interest rate equivalent to an annual rate of 8 %, capitalized quarterly?

Solution

First, you need to interpret the rate of 8 % as being nominal since it is accompanied by a capitalization period. A rate of 8 %, capitalized quarterly, represents in reality a biannual rate of 4 % ($\frac{8\,\%}{2} = 4\,\%$, semesters/year), if we let the interest capitalize. We therefore need to find i_{mon} knowing that $i_{bian} = 4\,\%$. According to the relation of rate equivalence, the identity

$$(1 + i_{mon})^{12} = (1 + i_{bian})^{2}$$

Must be satisfied. i_{mon} needs to be isolated*:*

$$(1 + i_{mon})^{12} = (1 + 4\%)^{2}$$
$$((1 + i_{mon})^{12})^{1/12} = ((1{,}04)^{2})^{1/12}$$
$$(1 + i_{mon})^{12/12} = (1{,}04)^{2/12}$$
$$1 + i_{mon} = (1{,}04)^{1/6}$$
$$i_{mon} = (1{,}04)^{1/6} - 1$$
$$i_{mon} = 0{,}6558\%$$

3. Exercise

Given an annual interest rate of 15 %. Find the periodic rates $i_{bian}, i_{mon,} i_{quart}$ that are equivalent.

(Answers : 7,23805 %; 3,55581 %; 1,17149 %)