

LINES AND SLOPES

Summary

1.	Elements of a line equation.....	1
2.	How to obtain a straight line equation	2
3.	Microeconomic applications	3
3.1.	Demand curve	3
3.2.	Elasticity problems	7
4.	Exercises	9

Required concepts for the courses : Micro-economic analysis, Managerial economy.

A **line** is a function that can be written in the following form: $y = mx + b$

In particular: graphically, a line is a function whose inclination constant at all points.

1. Elements of a line equation

The **slope**, represented by the letter ***m***, measures the inclination of the line. It corresponds to the variation of the value of ***y*** when ***x*** increases by one unit. Graphically, it represents the vertical variation of the line for a horizontal move of one positive unit.

If the line passes by the points (x_1, y_1) and (x_2, y_2) , the slope is obtained by the relation

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The ***Y-intercept***, represented by the letter ***b***, is the value of ***y*** when ***x*** is zero. It is the position of the line when it crosses the ***y***-axis.

Example

The equation $y = 3x - 4$ represents a straight line whose slope is 3 ($m = 3$) and whose y -intercept is -4 ($b = -4$).

Note that the x and y variables are arbitrary. They could have just as easily been named q and p , as it is the case in supply and demand curve. It is important to determine which of these variables constitutes the independent variable (which we place on the horizontal axis) and which constitutes the dependent variable (which we place on the vertical axis).

2. How to obtain a straight line equation

We will often need to find the equation of a straight line, given certain information. For example, what is the equation of the line passing points (1,4) and (2,8)? In order to answer this question, one needs to find the values of m and b that describe the straight line.

1. Determine the slope

By definition, the slope is measured by the relation

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of the line passing by (1,4) and (2,8) would be

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{2 - 1} = 4$$

which indicates that for a move of one unit to the right, there is a move of 4 units upwards. Also note that the choice of the "first" and the "second" point will not affect the calculation of the slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 8}{1 - 2} = 4$$

2. Find the y -intercept

In order to find the value of b , one must use a known point of the line and the slope we just determined:

We just calculated that $m = 4$. The equation of the present line is $y = mx + b = 4x + b$. We also know that the point (1,4) is located on this line and must therefore satisfy its equation :

$$4 = 4(1) + b \rightarrow 4 = 4 + b \rightarrow b = 0$$

Once again, the choice of the point used does not affect the outcome. Had we chosen the point (2,8), the calculation would have shown:

$$8 = 4(2) + b \rightarrow 8 = 8 + b \rightarrow b = 0$$

The slope and the y – intercept now being known, the line equation is $y = 4x + 0$ ou $y = 4x$.

3. Microeconomic applications

3.1. Demand curve:

Example 1

Let us assume that the demand curve is described by the following line $q = mp + b$. Find its equation given the following information: a promoter discovers that the demand for theater tickets is 1200 when the price is \$60, but decreases to 900 when the price is raised to \$75.

Solution :

The form of the equation $q = mp + b$ indicates that the price p , is the independent variable (like x), and the quantity q , is the dependent variable (like y). The problem allows us to deduce two points of the demand line: the points (60\$, 1200) and (75\$, 900). We must identify the slope and the y –intercept of the line.

Slope :

$$m = \frac{\Delta q}{\Delta p} = \frac{q_2 - q_1}{p_2 - p_1} = \frac{900 - 1200}{75 - 60} = \frac{-300}{15} = -20$$

The equation must therefore take on the following form: $q = -20p + b$. It is necessary to find the y -intercept using one of the two points.

y-intercept :

since (60\$, 1200) is a point on the demand curve, it must satisfy the following equation : $q = -20p + b$. By substitution, we obtain

$$1200 = -20(60) + b$$

$$1200 = -1200 + b$$

$$b = 2400$$

As a result, since $m = -20$ and $b = 2400$, the equation of the demand line is

$$q = -20p + 2400$$

It is interesting to note that once this line is found, we can evaluate what the demand is whatever the price. For example, the demand when the price is at \$40 would be obtained by calculating the variable q :

$$q = -20(40) + 2400$$

$$q = -800 + 2400$$

$$q = 1600$$

We could also obtain the price needed for a demand of 1000 tickets.

$$1000 = -20p + 2400$$

$$20p = 2400 - 1000$$

$$20p = 1400$$

$$p = 70 \$$$

Example 2

The equilibrium quantity and the equilibrium price of a product are determined by the point where the supply and demand curves intersect. For a given product, the supply is determined by the line

$$q_{supply} = 30p - 45$$

and for the same product, the demand is determined by the line

$$q_{demand} = -15p + 855.$$

Determine the price and the equilibrium quantity and trace the supply and demand curves on the same graph.

Solution :

We must determine the coordinates of point (q,p) , situated at the intersection of the two lines. This point must therefore satisfy both the supply and the demand equations. The solution to this problem is to solve :

$$\begin{aligned} q &= 30p - 45 \\ q &= -15p + 855 \end{aligned}$$

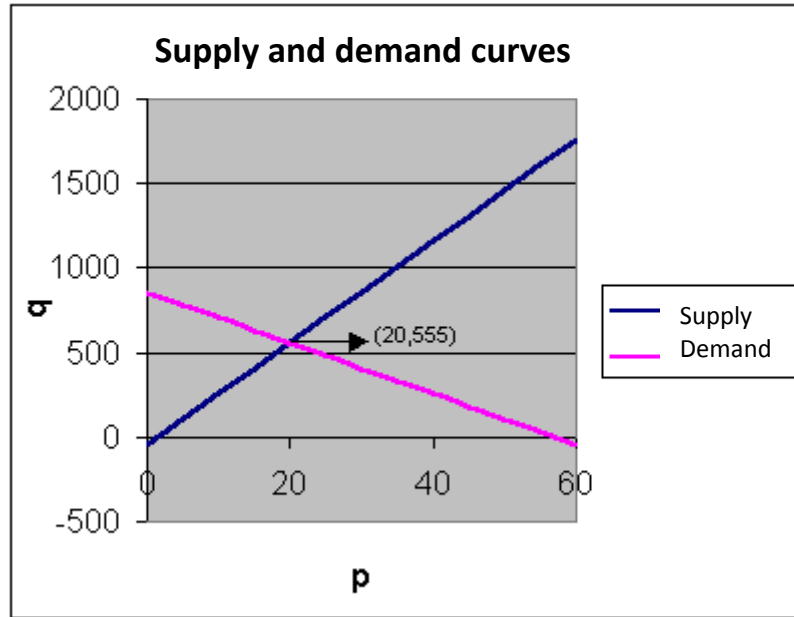
Thus,

$$\begin{aligned} 30p - 45 &= -15p + 855 \\ 45p &= 900 \\ p &= 20 \end{aligned}$$

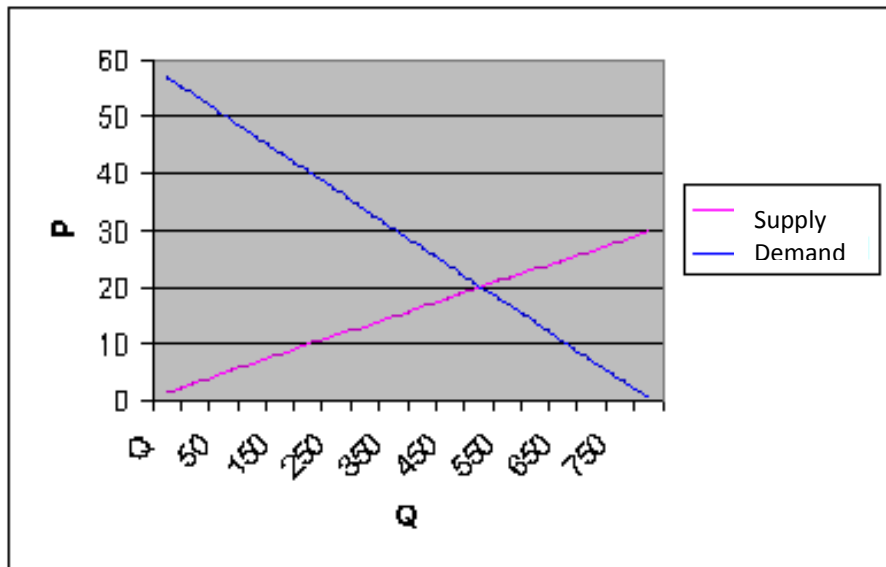
and

$$q = 30(20) - 45 = 555$$

The equilibrium price and quantity are therefore \$20 and 555.



In economics, it is usual to graphically represent the supply and demand curves by placing the price (p) on the ordinate and the quantity (q) on the abscissa.



3.2. Elasticity problems

A supply or demand problem sometimes gives as initial information only the equilibrium price and quantity and the price-elasticity. The latter measures the effect of a variation of the price on the supply or demand. We must remember that the price-elasticity is defined by the relation

$$\varepsilon_p = \frac{dq}{dp} \cdot \frac{p}{q}$$

Where the represented parameters are

p : price

q : quantity

$\frac{dq}{dp}$: derivative of the equation of the quantity with respect to the price

In a case where the price and the quantity of a product linearly depend on one another, we can use the average variation instead of the derivative, i.e.

$$\varepsilon_p = \frac{\Delta q}{\Delta p} \cdot \frac{p}{q}$$

If the price and quantity of a product linearly depend on each other, then the supply or demand function will have the form

$$q = mp + b,$$

where m is the slope of the line and is defined by

$$m = \frac{\Delta q}{\Delta p}$$

So, without knowledge of even two points of the line, we can still evaluate the slope if the price-elasticity coefficient is given.

$$\varepsilon_p = \frac{\Delta q}{\Delta p} \cdot \frac{p}{q} \rightarrow \varepsilon_p \cdot \frac{q}{p} = \frac{\Delta q}{\Delta p}$$

$$m = \frac{\Delta q}{\Delta p} = \varepsilon_p \cdot \frac{q}{p}$$

The value of the y -intercept, b , is obtained using all the remaining information.

Example

Find the supply function if the price-elasticity is 0,4 and the equilibrium price and quantity are \$500 and 200 units, respectively. Let us assume that the quantity depends linearly on the price.

Solution :

In order to find the equation of the line, we must find the slope. This is possible due to the relation between the slope and the elasticity that we established above:

$$m = \frac{\Delta q}{\Delta p} = \varepsilon_p \cdot \frac{q}{p}$$

$$m = 0,4 \cdot \frac{200}{500} = 0,4 \times 0,4$$

$$m = 0,16$$

All that is left to find is the value of b . The supply equation is $q = mp + b = 0,16p + b$. We also know that the point (500\$, 200) is located on this line and must thus satisfy the equation:

$$200 = 0,16(500) + b \rightarrow 200 = 80 + b \rightarrow b = 120$$

The supply curve equation is thus $q = 0,16p + 120$.

4. Exercises

Problem 1 :

A company produces shoes. When 30 shoes are produced, the total cost of production is \$325. When 50 shoes are produced, the costs increase to \$485. What is the cost equation (C) if it varies linearly in function to the number of shoes produced (q) ?

Solution : $C = 8q + 85$

Problem 2 :

Consider a market characterised by the following supply and demand curves :

$$\begin{aligned}q_{\text{demand}} &= -10p + 1000 \\q_{\text{supply}} &= 0,2p + 298,6\end{aligned}$$

Find the equilibrium price and quantity.

Solution : \$68,76 and 312,35

Problem 3 :

Find the demand function if the price-elasticity is -0.2 and the equilibrium price and quantity are \$100 and 2500 units, respectively. Supposing that the quantity depends linearly on the price.

Solution : $q = -5p + 3000$