Commitment and Nitpicky Behaviors in Insurance*

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Abstract

This paper investigates the economic mechanisms underlying nitpicky behaviors in insurance markets, with a focus on the insurer's inability to commit to a nitpicky strategy. Using a principal-agent framework, we analyze scenarios with and without commitment in a monopoly market. We find that when insurers can credibly commit, nitpicking disappears and full insurance coverage is reached in equilibrium. In contrast, non-commitment leads to the presence of nitpicking, partial insurance coverage, and welfare reductions. The robustness of these results is confirmed through extensions to a competitive market setting and cases involving insurance fraud. The study provides actionable insights for insurers and regulators, emphasizing the need for transparent contracts and automated claims processes to enhance market efficiency.

Keywords: Nitpicking; Commitment; Insurance fraud; Optimal contracts; Insurance market

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1 Introduction

Insurers' nitpicky behaviors, characterized by excessive scrutiny of claims and reductions in indemnity payments, have significant implications across insurance markets. These practices, while aimed at minimizing payouts, can impose severe financial stress on policyholders and erode trust in the insurance system. For instance, a 2022 survey revealed that 22% of insured individuals who sought reimbursement for emergency room or mental health services faced claim denials, leading to delayed resolutions and heightened financial strain.[1] Such behaviors compromise the perceived value of insurance, undermining its fundamental role as a tool for managing uncertainty and risk. This study investigates the economic mechanisms behind nitpicky behaviors in insurance, focusing on their origins, welfare implications, and potential policy remedies.

At the heart of this work lies the insurer's commitment problem, which serves as the primary driver of nitpicky behaviors. Specifically, suppose an insurer promises not to engage in nitpicky activities regardless of claim size. While this commitment may hold for smaller claims, where the costs of nitpicking exceed potential savings-a phenomenon commonly observed in industry practices, such as the use of "fast-track" AI technology to settle small indemnity claims within 30 seconds-it becomes less credible for larger claims, where the insurer has a strong incentive to deviate and reduce indemnity payments. This deviation undermines the insurer's credibility and leads to welfare-reducing outcomes.

This paper analyzes nitpicking under the commitment problem of the insurer within a principal-agent framework. We focus on a monopoly market setting with a risk-neutral insurer and a risk-averse policyholder. The insurer offers a contract that maximizes its expected profit subject to the policyholder's participation constraint. Upon a reported loss, the insurer may exert nitpicky effort to reduce its indemnity payment. We consider two scenarios: one where the insurer can credibly commit to a nitpicky strategy and another where it cannot. With commitment, the insurer determines an *a priori* nitpicky strategy before observing a loss. Without commitment, the insurer chooses an *a posteriori* nitpicky

[1] Source: see the link `https://www.kff.org/affordable-care-act/issue-brief/consumer-survey-highlights-problems-with-denied-health-insurance-claims/`.

strategy after a loss is reported. Our findings indicate that when commitment is feasible, nitpicking is never optimal, and the resulting equilibrium provides full insurance coverage. However, in the absence of commitment, nitpicking emerges as a cost-minimizing strategy for the insurer under certain conditions, leading to partial insurance coverage and a reduction in overall welfare.

The absence of commitment not only introduces overall inefficiencies but also results in Pareto-inefficient outcomes. Nitpicky behavior can be viewed as an overpriced gamble. The risk that the insurer may attempt to reduce indemnity payments diminishes the value of the insurance for policyholders. Consequently, policyholders prefer contracts with higher deductibles and lower premiums, which in turn reduce the insurer's equilibrium profit. These outcomes highlight the dual costs of commitment failure—diminished social welfare and reduced market efficiency.

To isolate the impact of nitpicking, we assume that the policyholder reports losses truthfully. This assumption enables us to disentangle the effects of nitpicking from potential exaggerations of losses by the policyholder. It also reflects real-world scenarios where loss assessments are inherently ambiguous, such as in cases of car accidents without witnesses or thefts with unverifiable valuations (Bourgeon & Picard 2014). Additionally, many policyholders lack financial literacy and may not fully understand contract terms, enabling insurers to interpret clauses in ways that favour their own interests (Peter & Ying 2020). Moreover, nitpicky behavior may be linked to the insurer's solvency conditions. Financially unstable insurers may delay, reduce, or deny claims to manage liquidity or avoid insolvency (Doherty & Schlesinger 1990). That said, we acknowledge that nitpicking may also be attributed to policyholders' *ex-post* moral hazard, wherein a policyholder might engage in insurance fraud by overreporting losses, or even reporting a loss which does not exist. However, data provided by Cheng et al. (2020) indicates that attributing nitpicking exclusively to insurance fraud may not be fully substantiated. There were $136,232$ P&C insurance complaints from 2005 to 2011 in U.S. and 72.9% of them are related to claim settlements,[2] and nearly 60% of these complaints resulted in

[2]The top three specific complaint types are delayed claims (31%), reduced settlement offer (20%), and denial of claim (11%).

consumer success. This high success rate suggests that nitpicking often targets honest claims rather than fraudulent ones.

To extend the robustness of our results, Section 5.1 incorporates *ex-post* moral hazard, where policyholders may engage in fraudulent reporting. Extending the framework considered in Bourgeon & Picard (2014), we examine a setting where the insurer can commit to an audit strategy but not to a nitpicky strategy. While Bourgeon & Picard (2014) found that both fraud and nitpicking disappear in equilibrium, our analysis suggests that fraud is eliminated but nitpicking persists. Moreover, in Section 5.2, we consider a perfectly competitive insurance market, showing that commitment failures lead to similar outcomes even under free market conditions.

The findings of this study highlight the importance of commitment in addressing insurer nitpicky practices. When insurers can credibly commit to predefined claim-handling strategies, nitpicking disappears. Policymakers can draw actionable insights from this conclusion to enhance fairness and efficiency in insurance markets. First, regulators should enforce greater clarity and transparency in insurance contracts by standardizing terms and clearly defining claim-handling procedures. Additionally, technologies like blockchain and smart contracts could automate the claims process, ensuring pre-agreed indemnity payments without discretionary alterations. Second, regulatory mechanisms to audit claim-handling patterns and penalize insurers with high rates of claim reductions or delays could discourage nitpicky behavior. In addition, educating policyholders about their rights and the claims process can further mitigate exploitative practices.

This study contributes to the literature on hidden actions arising from asymmetric information, where agents' unobservable behavior creates inefficiencies. Hidden action has been widely studied across contexts such as efficient wage contracts (Zhu 2018), managerial incentive schemes (Bolton & Dewatripont 2004), optimal debt financing (Innes 1990), team production (Holmstrom 1982), and ex-ante moral hazard in insurance markets (Holzapfel et al. 2024). Particularly, this work is closely related to policyholder's *ex-post* moral hazard, which is a specific form of hidden action. This concept refers to a situation in which policyholders misreport or even defraud after the occurrence of a loss. For instance, Picard (1996) examines insurance fraud and finds that insurers' auditing strategies can fully (partially) prevent fraud with (without) auditing commitment. Similarly, Boyer (2003) and Boyer (2004) explore loss misreporting, demonstrating that overcompensation for small (large) claims helps mitigate opportunistic reporting, with (without) auditing commitment. Boyer & Peter (2020) further investigate insurance fraud in the context of adverse selection. Despite these connections, our focus differs fundamentally: while *ex-post* moral hazard involves hidden actions by the policyholder, we concentrate on nitpicking, which entails hidden actions undertaken by the insurer.

Second, this paper relates to the literature on nitpicking behaviors of insurers, a concept first introduced by Bourgeon & Picard (2014). While we adopt the term "nitpicking", similar behaviors have been studied under various terms, including "contract non-performance" (Peter & Ying 2020), "dishonest practices" (Siemering 2021), and "default risk" of insurers (Doherty & Schlesinger 1990). These studies highlight the significant impact of such behaviors on insurance demand. For instance, Doherty & Schlesinger (1990) and Schlesinger & vd Schulenburg (1987) analyzed how the presence of exogenous default risk influences policyholders' insurance purchasing decisions. They found that risk-averse policyholders are unlikely to opt for full insurance coverage, even when premiums are actuarially fair. Subsequent research has built on the seminal work of Doherty & Schlesinger (1990), incorporating additional factors into the analysis while reaching similar conclusions. These extensions include consideration of partial default (Briys et al. 1991, Mahul & Wright 2007), divergent beliefs regarding default risk (Cummins & Mahul 2003), continuously distributed loss amounts (Meyer & Meyer 2010), ambiguity aversion (Peter & Ying 2020, Biener et al. 2019), and the relationship between default probabilities and loss amounts (Bernard & Ludkovski 2012).

This paper contributes to the literature in three ways. First, to our knowledge, this paper is the first to endogenize insurers' costly nitpicky behavior as a consequence of a commitment problem. Prior research typically treats nitpicking as an exogenous factor and examines its impact on insurance demand. While Bourgeon & Picard (2014) and Bourgeon & Picard (2020)[3] do endogenize nitpicking, they implicitly assume it to be

[3] Bourgeon & Picard (2020) explore the interaction between incomplete contracts and legal frameworks,

costless and committed. Second, this study analyzes endogenous nitpicky strategies across a continuum of loss amounts, moving beyond the binary loss states typically considered in existing studies. Third, we extend the model to explore equilibrium outcomes and social welfare under different market structures, including both monopoly and perfect competition. This analysis provides a broader perspective compared to the primarily competitive settings explored in previous research.

The structure of the paper is as follows. Section 2 presents a principal-agent framework for analyzing nitpicking behavior. Section 3 investigates optimal insurance contracts under both commitment and non-commitment scenarios. Section 4 conducts a welfare analysis, examines Pareto efficiency, and provides comparative statics of the equilibria. Section 5 extends the analysis to two additional contexts: insurance fraud and perfect competition. Finally, Section 6 concludes. All proofs are provided in the appendices.

2 Model Framework

In this section, we present the model that captures insurers' nitpicking behavior within a principal-agent framework. Our model explores how insurers chooses their nitpicky strategy based on the contract structure—specifically, whether they can commit *a priori* to a nitpicky strategy. The model also examines how the nitpicky behavior varies with the realized loss amount and impacts the insurer's profitability and the policyholder's expected utility. We note that existing studies, such as Bolton & Dewatripont (2004), may use terms like "observability" and "information symmetry" (or their opposites, "unobservability" and "information asymmetry") to describe concepts analogous to "commitment" and "non-commitment" as used in this paper.

2.1 Model setting

For the main part of the analysis, we consider a monopoly market where a representative insurer offers coverage for a potential loss to a representative risk-averse policyholder.

emphasizing their role in balancing insurer discretion and policyholder protection.

Analysis under perfect competition is considered in Section 5.2. The loss amount L is a random variable representing potential losses and follows a distribution with cumulative distribution function (CDF) $F_L(\ell) = \Pr(L \leq \ell)$, where ℓ denotes a specific reported value of the loss. We consider $L \in (0, \overline{\ell}]$ where $\overline{\ell} < +\infty$ is the maximum possible loss.[4]

The insurance premium is denoted by p, and the deductible by $d \in [0, \overline{\ell}]$, such that the insurance contract covers losses exceeding d. We further assume that, should a loss occur, the policyholder will always truthfully reports the loss value ℓ to the insurer.[5] Upon a reported loss ℓ, the insurer exerts a nitpicky level, denoted by $\epsilon(\ell)$. A nitpicky strategy refers to a specific functional form of $\epsilon(\cdot)$, and its determination involves specifying this function. For simplicity of exposition, we will omit the argument ℓ in subsequent discussions and refer to the nitpicky strategy simply as ϵ when no ambiguity or confusions arise. Moreover, let $e := \epsilon(\ell)$ denote a specific nitpicky level, which is the value of $\epsilon(\ell)$ for some $\epsilon(\cdot)$ and loss ℓ. As discussed in Section 4, the determination of the optimal nitpicky strategy depends on the insurer's ability to commit. In the case of commitment, the function ϵ is decided *a priori*—before the loss ℓ is observed. Without commitment, ϵ is chosen *a posteriori*—after ℓ is reported—allowing the insurer to adjust its *ex-post* nitpicky effort based on the realized loss.

To clarify this distinction, consider a simple example. Suppose, at the time of selling the contract, the insurer commits to a fixed nitpicky level $\tilde{e}$ irrespective of the reported loss. If, after the sale, a small loss $\tilde{\ell}$, which is lower than the cost of exerting $\tilde{e}$ is reported, the insurer must still exert $\tilde{e}$ if it is committed, even though this effort is suboptimal. In contrast, without commitment, the insurer can deviate from the *a priori* strategy, reducing its nitpicky level, or even opting for no nitpicking if it is deemed unnecessary.

By applying nitpicking, the insurer can *randomly reduce a fraction of the indemnity.* Let $Z_e \in [0, 1]$ represent the random fraction of the contractual indemnity $(\ell - d)_+$ that is cut by the insurer given a nitpick level e. The actual indemnity paid to the policyholder

[4]Mathematically, we assume $\mathrm{co}\big((\mathrm{supp}(F_L)\big) = (0, \overline{\ell}]$ with $\overline{\ell} < +\infty$, where $\mathrm{co}(\cdot)$ denotes the convex hull of a set.

[5]Extension of the benchmark framework to incorporating insurance fraud, i.e. untruthfully reporting a loss to the insurer, is discussed in Section 5.1.

is then given by:

$$(1 - Z_e)(\ell - d)_+,$$

where $Z_e \sim F_{Z_e}(z)$ is drawn from a distribution that depends on e. The indemnity cut is modeled as a random variable here to reflect the inherent uncertainty in the claim-handling process. Besides the nitpicky level e, latent factors such as policyholder characteristics, limitations in financial literacy, and specific circumstances of the loss influence the indemnity cut. These factors are incorporated into the conditional distribution $F_{Z_e}(z)$.

Following Bourgeon & Picard (2014), we assume that, for any nitpicky strategy $\epsilon(\cdot)$ and reported loss, the *expected fraction of the indemnity cut* is equal to the nitpicky level, i.e.,

$$\epsilon(\ell) = \mathbb{E}[Z_{\epsilon(\ell)}] = \int_0^1 z\, dF_{Z_{\epsilon(\ell)}}(z) \in [0, 1].$$

This implies that, on average, the indemnity cut is proportional to the chosen nitpicky level. Table 1 summarizes the relationship between the nitpicky level, the indemnity cut, and the actual indemnity paid. When $e = 0$, the indemnity cut $Z_e = 0$[6], meaning the full contractual indemnity is paid. In the case where $e = 1$, $Z_e = 1$, so the insurer fully denies the claim, resulting in no indemnity payment. For values of e between 0 and 1, the actual indemnity is randomly reduced by the fraction $Z_e \in [0, 1]$ based on $F_{Z_e}(z)$. Finally, we remark that the indemnity cut depends on the reported loss. Given a nitpicky strategy $\epsilon(\cdot)$, the nitpicky level is a function of ℓ. Hence, the indemnity cut can be equivalently written as $Z_{\epsilon(\ell)}$, which is a random variable that varies with ℓ.

	Circumstance	Nitpicky level	Indemnity cut	Actual indemnity
(1)	Full indemnity	$e = 0$	$Z_e = 0$	$(\ell - d)_+$
(2)	No indemnity	$e = 1$	$Z_e = 1$	0
(3)	Random indemnity	$e \in (0, 1)$	$Z_e \in [0, 1]$	$(1 - Z_e)(\ell - d)_+$

Table 1: Nitpicky level, indemnity cut, and actual indemnity

[6]That is, $\Pr(Z_e = 0) = 1$. Unless otherwise specified, all equalities involving random variables throughout this paper should be interpreted as holding almost surely.

Insurer's Cost of Nitpicking and Expected Profit

Engaging in nitpicking incurs costs for the insurer. These costs, denoted by $c(e)$, include auditing, investigation, negotiation, and reputational costs. We assume the cost function satisfies the properties $c(0) = 0$, $c'(e) > 0$, $c''(e) > 0$ for $e \in [0, 1]$ and $c'(1) = +\infty$[7], that is, the marginal cost of nitpicking is positive and increases with the nitpicky level. In addition, for the cost function to be well-defined, we let $c'(0) = \lim_{e \to 0^+} c'(e)$. These assumptions imply that the cost of nitpicking grows rapidly as e approaches 1, which is a desirable feature since denying all claims (i.e., $\epsilon(\ell) = 1$ for all ℓ) would be excessively costly and unreasonable in practice.

The insurer's expected profit $\pi(p, d, \epsilon)$ is determined by the premium p, the deductible d, and a nitpicky strategy ϵ:

$$
\begin{aligned}
\pi(p, d, \epsilon) &= p - \theta \mathbb{E}\big[c(\epsilon)\big] - \theta \mathbb{E}\Big[(L-d)_+ \Big(1 - Z_\epsilon\Big)\Big] \\
&= p - \theta \mathbb{E}\big[c(\epsilon)\big] - \theta \mathbb{E}\Big[\mathbb{E}\Big[(L-d)_+ \Big(1 - Z_\epsilon\Big) | L\Big]\Big] \\
&= p - \theta \mathbb{E}\big[c(\epsilon)\big] - \theta \mathbb{E}\Big[(L-d)_+ \big(1 - \epsilon\big)\Big].
\end{aligned} \tag{1}
$$

In Equation (1), θ is the probability of a loss occurring; $\theta \mathbb{E}\left[c\left(\epsilon\right)\right]$ captures the *expected nitpicking cost*; and the last term, $\theta \mathbb{E}\left[(L-d)_+ \left(1-\epsilon\right)\right]$, accounts for the *expected actual indemnity payment* after nitpicking.

Policyholder's Expected Utility

The risk-averse policyholder is assumed to be endowed with an initial wealth w, and risk preference represented by a *Von Neumann-Morgenstern utility function* $v(\cdot)$, which is twice continuously differentiable, with $v'(w) > 0$ and $v''(w) < 0$. The expected utility of the policyholder, denoted by $u(p, d, \epsilon)$, is given by:

$$
u(p, d, \epsilon) = (1-\theta) v(w-p) + \theta \mathbb{E}\Big[v\Big(w - p - L + (1 - Z_\epsilon)(L-d)_+\Big)\Big],
$$

[7] $c'(1) = +\infty$ could be relaxed at the cost of increased mathematical complexity.

where $v(w-p)$ is the utility when no loss occurs, and $\mathbb{E}\left[v\Big(w-p-L+(1-Z_\epsilon)(L-d)_+\Big)\right]$ represents the expected utility when a loss happens, considering the actual indemnity received after nitpicking.

2.2 Sequence of play

We model the strategic interaction between the insurer and the policyholder as a sequential-move game (see Figure 1), where each party's decisions unfold in stages. An illustrative

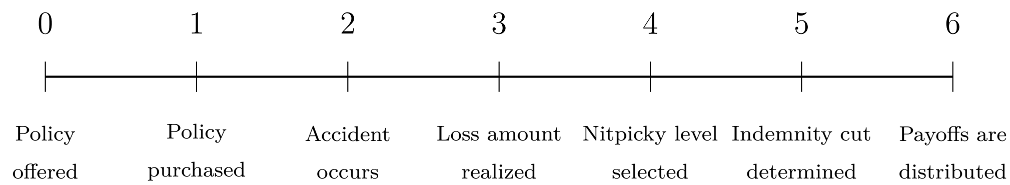

Figure 1: Sequence of play

game tree associated with the sequence of play is presented in Figure A1. In addition, let u_0 be the reservation utility for the policyholder. The reservation utility is the threshold for the policyholder to accept the insurance contract. We assume that it is bounded below by the utility of being uninsured:

$$u_0 \geq (1-\theta)v(w) + \theta\mathbb{E}[v(w-L)].$$

The sequence of decisions and outcomes in this game proceeds as follows:

Stage 0 (Insurance Policy Offer): The insurer decides whether to offer an insurance contract (p, d, ϵ) based on expected profitability. If the insurer anticipates a non-negative expected profit, i.e., $\pi(p, d, \epsilon) \geq 0$, the contract is offered, and we proceed to the next stage. Conversely, if the expected profit is negative, then the insurer chooses not to offer the contract. In this case, the payoff of both parties is $(0, u_0)$, meaning the insurer earns nothing and the policyholder retains her reservation utility u_0.

Stage 1 (Insurance Policy Purchase): The policyholder decides whether to accept or reject the contract based on her expected utility. If the expected utility with insurance exceeds the reservation utility, i.e., $u(p, d, \epsilon) \geq u_0$, the policyholder accepts the contract, and the game moves to the next stage. If the expected utility is lower than the reservation utility, then the policyholder rejects the contract, resulting in a payoff of $(0, u_0)$.

Stage 2 (Loss Occurrence): Nature determines whether a loss occurs, with probability θ. If no loss occurs, the game ends with the payoff $\big(p, v(w-p)\big)$, where the insurer collects the premium p without paying an indemnity, and the policyholder receives the utility level $v(w-p)$. If a loss does occur, the game proceeds to the next stage.

Stage 3 (Loss Realization): If a loss occurs, nature randomly selects a loss amount $L = \ell$ from the loss distribution $F_L(\ell)$.

Stage 4 (Insurer's Nitpicky Level Choice): After observing the loss, the insurer chooses a nitpicky level e based on the nitpicky strategy ϵ (selected *a priori* or *a posteriori*) to apply in assessing the claim.

Stage 5 (Indemnity Cut Determination): After the insurer selects a nitpicky level e, nature determines the actual indemnity cut z, drawn from the distribution $F_{Z_e}(z)$. The actual indemnity paid to the policyholder is then $(\ell - d)_+(1 - z)$.

Stage 6 (Payoff Distributed): Finally, the actual reduced indemnity, together with the nitpicky cost, results in the final payoff:

$$\Big(p - (\ell - d)_+(1 - z) - c(e),\ v\big(w - p - \ell + (\ell - d)_+(1 - z)\big)\Big).$$

Here, the first term represents the insurer's financial outcome after paying out the indemnity and incurring the cost of nitpicking $c(e)$. The second term denotes the policyholder's utility after paying the premium p, suffering the loss ℓ, and receiving a reduced indemnity.

In this game, the policyholder's decision-making is shaped by their expectations regarding the insurer's nitpicky strategy. The equilibrium outcome is contingent on whether the insurer can credibly commit to a predetermined nitpicky strategy. The subsequent sections provide a detailed analysis of the contract structure and examine the implications of the insurer's ability to commit.

3 Optimal Contracts

This section presents the optimal insurance contracts for two scenarios: when the insurer can commit to a nitpicky strategy and when it cannot. In both scenarios, the nitpicky level e could depend on the reported loss ℓ. The fundamental difference here is that, if the insurer can commit, the nitpicky strategy ϵ is determined *before* the loss ℓ is observed. Conversely, if the insurer cannot commit, the nitpicky level is determined *after* observing ℓ, and the policyholder must infer the nitpicky strategy, assuming the insurer seeks to maximize its *ex-post* profit.

3.1 Committed Nitpicky Strategy

When the insurer can credibly commit to a nitpicky strategy, the policyholder can accurately anticipate the insurer's actions regarding claims. This situation may arise in contexts such as InsurTech platforms that offer clearly defined claim review standards, which are transparent and accessible to policyholders. Another relevant situation is that public legal disputes between insurers and policyholders can promote an environment where the insurer's practices are subject to external scrutiny, thereby enhancing transparency.

Formally, the insurer designs an insurance contract (p, d, ϵ) to maximize its expected profit. Under this framework, the nitpicky strategy ϵ is determined at the inception of the contract. The insurer's optimization problem is thus characterized as follows:

$$
\begin{aligned}
&\sup_{p,d,\epsilon} \ \pi(p, d, \epsilon) \qquad (2) \\
\text{s.t.} \quad & u(p, d, \epsilon) \geq u_0, \\
& p \geq 0, 0 \leq d \leq \bar{\ell}, 0 \leq \epsilon \leq 1, \quad \forall \ell \in (0, \bar{\ell}].
\end{aligned}
$$

The solution to Problem (2) shows that when the insurer is able to commit, it has no incentive to engage in nitpicking. The optimal contract in this case is referred to as the "first-best" insurance contract, denoted by $(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}})$, where the superscript c stands for "commitment". The solution is summarized in Proposition 1.

Proposition 1 (Optimal Contract with Commitment) *When the insurer is able to commit to a predetermined nitpicky strategy, the first-best insurance contract* (p^c, d^c, ϵ^c) *is characterized as follows:*

(1) If $u_0 \leq v(w)$*, then*

$$\epsilon^c = 0, \quad d^c = 0, \quad p^c = w - v^{-1}(u_0), \quad \forall \ell \in (0, \bar{\ell}].$$

(2) If $u_0 > v(w)$*, no feasible solutions exist.*

Proof: The proof is provided in Appendix B. ■

This proposition provides two main insights. First, when $u_0 > v(w)$, the participation constraint for the policyholder cannot be satisfied. Specifically, the participation constraint requires that the insurer offers a contract satisfying $u(p^c, d^c, \epsilon^c) \geq u_0 > v(w)$. Recall that $v(w)$ is the policyholder's utility when no loss occurs. For the insurer to meet this condition, it would need to provide a contract more favorable than the most generous possible contract—full insurance, no premium, and no nitpicking in any case. Since no feasible contract can surpass this level of generosity, satisfying the participation constraint in this case becomes impossible.

On the other hand, when $u_0 \leq v(w)$, there exists a feasible contract that satisfies the policyholder's participation constraint. The economic rationale underlying the insurer's strategy becomes evident in this context: although nitpicking reduces indemnity payouts, it introduces additional costs and uncertainty, discouraging the policyholder from accepting the contract. To minimize these negative effects, the insurer refrains from nitpicking, resulting in $\epsilon^c = 0$. Given that risk-averse policyholders generally prefer full insurance (Mossin 1968), the optimal contract under these conditions offers full coverage ($d^c = 0$) at an "indifferent" premium level, which is the premium at which the policyholder is indifferent between accepting or rejecting the contract:

$$u(p^c, d^c = 0, \epsilon^c = 0) = v(w - p^c) = u_0, \ \ \forall \ell \in (0, \bar{\ell}].$$

The formula above results in $p^c = w - v^{-1}(u_0)$.

3.2 Uncommitted Nitpicking Strategy

When the insurer is unable to commit to a nitpicky strategy, the optimal nitpicky strategy $\epsilon(\ell)$ will be determined strategically according to the reported loss, $L = \ell$. In this case, given a loss ℓ, the optimal nitpicky level is the solution to the following problem:

$$e^* := \epsilon^*(\ell) \in \arg\max_{0 \le e \le 1} \big\{ p - (\ell - d)_+(1 - e) - c(e) \big\}, \ \forall \ \ell \in (0, \overline{\ell}]. \tag{3}$$

Since the cost function $c(e)$ is convex and monotonic, it can be shown that the *ex post* optimal nitpicky strategy ϵ^*, given p and d, is uniquely determined by the following equation, :

$$\epsilon^*(\ell) \begin{cases} = 0, & \text{if } (\ell - d)_+ \le c'(0), \\ > 0 \text{ s.t. } c'(\epsilon^*(\ell)) = (\ell - d)_+, & \text{if } c'(0) < (\ell - d)_+ \le c'(1), \\ = 1, & \text{if } (\ell - d)_+ > c'(1). \end{cases} \tag{4}$$

In other words, depending on the marginal nitpicky cost, $c'(\cdot)$, and its choice of deductible, d, the insurer could spend positive nitpicky effort if the reported loss ℓ satisfies $c'(0) < (\ell - d)_+ \le c'(1)$. The equilibrium nitpicky strategy without commitment, e^{wc} is then obtained by solving Equation (3) with d and p set at the optimal value. Here, the superscript wc stands for "without commitment". The following lemma provides the characteristics of ϵ^*, which are essential for subsequent analysis.

Lemma 1 *Given the reported loss amount ℓ, the maximizer e^* of Equation* (3) *is unique, continuous, and non-increasing with respect to all $d \in [0, \overline{\ell}]$.*

Proof: The proof is provided in Appendix C. ■

Figure 2 provides a graphical representation of the optimal nitpicky level e^* over different values of deductibles. The functional specifications used to generate this figure are discussed in Section 4.3. When $(\ell - d)_+ \le c'(0)$, the marginal benefit of nitpicking is insufficient to offset its cost, so the insurer finds no incentive to engage in nitpicking. However, when $(\ell - d)_+ > c'(0)$, nitpicking becomes economically beneficial, and the insurer engages in a positive level of nitpicking. Additionally, when $(\ell - d)_+ > c'(1)$—a case not illustrated in Figure 2— e^* is set to be 1, as it would be unreasonable for the insurer to exceed this level of indemnity reduction. Nonetheless, we note that this case is generally unlikely to occur since $c'(1)$ is expected to be quite large in practice.

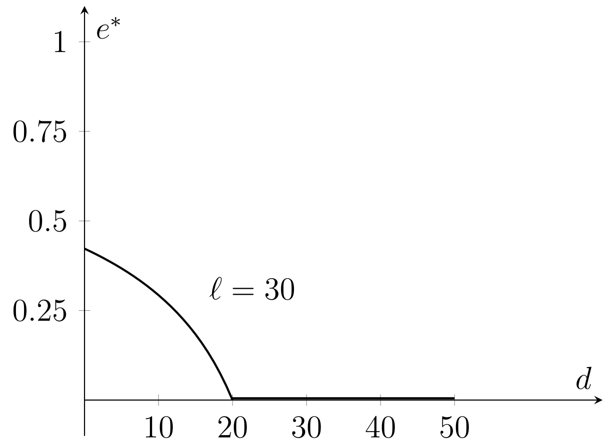

Figure 2: Optimal nitpicky level as a function of the deductible

In the case without commitment, the rational policyholder understands that the insurer will determine an optimal nitpicky level by solving Equation (3) after observing ℓ. The policyholder incorporates this expectation into their decision-making process when evaluating whether to accept or reject the insurance contract. Consequently, the insurer's optimal nitpicky level aligns perfectly with the policyholder's expectations. We refer to the equilibrium contract under the non-commitment scenario as the "second-best" insurance contract, denoted by $(p^{\text{wc}}, d^{\text{wc}}, \epsilon^{\text{wc}})$, which solves the following optimization problem:

$$
\begin{aligned}
& \sup_{p,d,\epsilon} \; \pi(p,d,\epsilon) \\
\text{s.t.} \quad & u(p,d,\epsilon) \geq u_0, \\
& \epsilon \text{ determined by Equation (4)}, \\
& p \geq 0, 0 \leq d \leq \overline{\ell}.
\end{aligned} \tag{5}
$$

The solution to Problem (5) demonstrates that some degree of nitpicking may be optimal when the insurer cannot commit to a predetermined nitpicky strategy. To aid our analysis, we define a threshold for the reservation utility, denoted by u_t, as follows:

$$
u_t = \sup_{p,d,\epsilon} \; u(p,d,\epsilon), \tag{6}
$$

$$\text{s.t.} \quad \epsilon \text{ determined by Equation (4)},$$
$$p \geq 0, 0 \leq d \leq \bar{\ell}.$$

The optimal contracts in the absence of commitment is summarized in the following proposition.

Proposition 2 (Optimal Contract without Commitment) *When the insurer is not able to commit to a predetermined nitpicky strategy, there exists a threshold $u_t \leq v(w)$ such that:*

1. *If $u_0 \leq u_t$, a second-best insurance contract $(p^{wc}, d^{wc}, \epsilon^{wc})$ exists and is characterized as follows:*

   (a) When $c'(0) < \bar{\ell} - d^{wc}$, there exists a threshold $\hat{\ell} \in (0, \bar{\ell}]$ such that $\epsilon^{wc} > 0$ for all $\ell \in [\hat{\ell}, \bar{\ell}]$. The optimal nitpicky strategy is uniquely defined by the equation:

   $$(\ell - d^{wc})_+ = c'(\epsilon^{wc}) \quad \text{for all } \ell \in [\hat{\ell}, \bar{\ell}],$$

   and the premium p^{wc} is uniquely determined by the condition:

   $$u(p^{wc}, d^{wc}, \epsilon^{wc}) = u_0.$$

   (b) When $c'(0) \geq \bar{\ell} - d^{wc}$, the nitpicky level is zero for all $\ell \in (0, \bar{\ell}]$, i.e., $\epsilon^{wc} = 0$ for all ℓ. In this case, the premium and deductible are given by $p^{wc} = w - v^{-1}(u_0)$ and $d^{wc} = 0$, respectively.

2. *If $u_0 > u_t$, no feasible solutions exist.*

Proof: The proof is provided in Appendix D. ∎

We provide a brief interpretation of this proposition. First, consider the infeasibility result: from Problems (5) and (6), u_t represents the highest utility attainable by the policyholder if she could determine the contract terms. As a result, no feasible contract can satisfy $u(p, d, \epsilon) > u_t$. Second, we examine the economic rationale behind the feasible solutions. Without commitment, the insurer may opt for a positive nitpicky level upon observing a large ℓ, provided the marginal cost of nitpicking is lower than its marginal benefit (i.e., $c'(0) < \ell - d^{\text{wc}}$). In such cases, engaging in nitpicking could yield higher profits for the insurer than abstaining from it.

Furthermore, both the insurance coverage and premium are lower in the absence of commitment (i.e. $d^{\text{wc}} \geq d^{\text{c}}$ and $p^{\text{wc}} \leq p^{\text{c}}$). The first argument is straightforward, as $d^{\text{wc}} \geq d^{\text{c}} = 0$. The second argument follows directly from the equality $u(p^{\text{wc}}, d^{\text{wc}}, \epsilon^{\text{wc}}) = u(p^{\text{c}}, d^{\text{c}}, \epsilon^{\text{c}}) = u_0$, which implies the following condition:

$$v(w - p^{\text{c}}) = (1-\theta)v(w - p^{\text{wc}}) + \theta\mathbb{E}\big[v\big(w - p^{\text{wc}} - L + (L - d^{\text{wc}})_+(1 - Z_\epsilon)\big)\big] \leq v(w - p^{\text{wc}}).$$

Due to the monotonicity of $v(\cdot)$, it follows that $p^{\text{wc}} \leq p^{\text{c}}$. This inequality is strict when $c'(0) < \bar{\ell} - d^{\text{wc}}$, i.e., when there is a positive nitpicky level.

Finally, we remark that the insurance market is more likely to collapse in the absence of commitment. The infeasibility threshold u_t is lower in this case compared to the commitment scenario, where the threshold is $v(w)$). This is because, for any $p \geq 0$, $0 \leq d \leq \bar{\ell}$ and ϵ such that $0 \leq e \leq 1, \forall \ell$, it holds that:

$$u(p, d, \epsilon) \leq= v(w) \text{ for any } \ell.$$

Thus, the insurance market is more prone to to collapse when the insurer cannot commit to its nitpicky strategy. For instance, if $u_t < u_0 \leq v(w)$, feasible contracts exist under commitment but become unviable in the absence of commitment.

4 Equilibrium Analysis

We now investigate the Pareto efficiency and welfare implications of equilibrium outcomes and the sensitivity of these outcomes to different factors. Numerical illustrations of equilibrium outcomes are also provided.

4.1 Pareto Efficiency and Social Welfare

This subsection examines the Pareto efficiency of equilibrium outcomes. Pareto efficiency refers to a state in which no party's situation can be improved without worsening the other party's condition. According to Arrow (1951), under certain regular conditions—one of which is committed product qualities—every market equilibrium is Pareto-efficient. The following proposition extends this insight to the insurance context studied.

Proposition 3 (Pareto Efficiency) *For $u_0 \leq u_t$, the Pareto efficiency of equilibrium outcomes is as follows:*

1. *If $c'(0) \geq \bar{\ell} - d^{wc}$, both (p^c, d^c, ϵ^c) and $(p^{wc}, d^{wc}, \epsilon^{wc})$ are Pareto-efficient.*

2. *If $c'(0) < \bar{\ell} - d^{wc}$, only (p^c, d^c, ϵ^c) is Pareto-efficient.*

Proof: The proof is provided in Appendix E. ■

The equilibrium contract is always Pareto-efficient when the insurer can commit to a nitpicky strategy. In this case, the insurer opts *not* to engage in nitpicking (see Proposition 1). However, when the insurer cannot commit, Pareto efficiency depends on the nitpicking cost. If the cost is sufficiently high (i.e., $c'(0) \geq \bar{\ell} - d^{\mathrm{wc}}$), the equilibrium remains Pareto-efficient because the insurer refrains from nitpicking. Conversely, if the cost is low (i.e., $c'(0) < \bar{\ell} - d^{\mathrm{wc}}$), the equilibrium becomes Pareto-inefficient as the insurer's engagement in nitpicking introduces inefficiencies.[8] These results suggest that if the insurer can credibly signal that the cost of nitpicking is prohibitively high (i.e., $c'(0) \geq \bar{\ell} - d^{\mathrm{wc}}$), both parties are incentivized to cooperate, ensuring a Pareto-efficient outcome.

We now examine the social welfare implications of nitpicking. The following proposition shows that both parties are better off when the insurer can commit.

Proposition 4 (Social Welfare) *For $u_0 \leq u_t$ and all $\ell \in (0, \bar{\ell}]$, the welfare of both parties is as follows:*

1. *The policyholder achieves the same utility level in both cases,*

$$u(p^c, d^c, \epsilon^c) = u(p^{wc}, d^{wc}, \epsilon^{wc}) = u_0.$$

[8]Strictly speaking, this scenario should be characterized as "unconstrained Pareto-inefficient" because the construction of a Pareto-improving allocation here does not account for incentive compatibility (i.e., $\epsilon = \epsilon^{\mathrm{wc}}$). However, following the terminology used in Cabrales et al. (2003), we adopt the term "Pareto-inefficient" in this context.

2. The insurer earns a higher profit when it can commit:

$$\pi(p^c, d^c, \epsilon^c) \geq \pi(p^{wc}, d^{wc}, \epsilon^{wc}),$$

with the inequality being strict if and only if nitpicking occurs, i.e., $c'(0) < \bar{\ell} - d^{wc}$.

Proof: The proof is provided in Appendix F. ■

Contrary to existing studies which often suggest that the absence of commitment primarily harms the policyholder without affecting the insurer (Picard 1996), Proposition 4 shows a different dynamic. In our model, the inability to commit introduces nitpicking, which impacts the insurer by incurring nitpicking costs and reducing profitability. This divergence stems from the market structure assumed in our analysis. Unlike the competitive market setting in prior studies, where the policyholder holds a dominant position, our model assumes a monopoly market in which the insurer is the dominant party. In this case, the policyholder's equilibrium expected utility always equals her reservation utility.

This distinction emphasizes the critical role of market structure in determining the welfare implications of contract design. Specifically, our findings indicate that the dominant party (the insurer, in this case) is more susceptible to the negative consequences of an inability to commit to the nitpicky strategy. Section 5.2 discusses the optimal contracts under perfect competition. In this case, it can be shown that the policyholder's expected utility is reduced in the absence of commitment.

4.2 Comparative Statics

This subsection examines the factors that influence the equilibrium outcome. Proposition 1 establishes that when the insurer can commit, the equilibrium expected profit depends exclusively on the reservation utility level u_0 and is unaffected by the distribution of the indemnity cut $F_{Z_e}(z)$ or the nitpicking cost $c(e)$. In contrast, when the insurer cannot commit, its equilibrium profit is impacted by all these factors. The following proposition formalizes these relationships.

Proposition 5 (Comparative Statics) *For any nitpicky level e and loss ℓ, let $F_{Z_e^\Lambda}(z)$ and $F_{Z_e^\lambda}(z)$ be two distinct CDFs of the random indemnity cut Z such that*

$$\int_0^t F_{Z_e^\lambda}(z)\,dz \leq \int_0^t F_{Z_e^\Lambda}(z)\,dz, \quad \forall t \in [0,1],$$

with $\mathbb{E}[Z_e^\lambda] = \mathbb{E}[Z_e^\Lambda] = e$. Additionally, let $c^\kappa(\cdot)$ and $c^K(\cdot)$ be two cost functions such that $c^K(e) \geq c^\kappa(e)$. When $u_0 \leq u_t$, the following results hold:

1. *When the insurer cannot commit to a nitpicky strategy, its equilibrium expected profit decreases (all else being equal) with:*

   - *Increased volatility of the indemnity cut:*

   $$\pi^\Lambda(p_\Lambda^{wc}, d_\Lambda^{wc}, \epsilon_\Lambda^{wc}) \leq \pi^\lambda(p_\lambda^{wc}, d_\lambda^{wc}, \epsilon_\lambda^{wc}),$$

   where $\pi^j(p_j^{wc}, d_j^{wc}, \epsilon_j^{wc})$ is the insurer's equilibrium profit given $F_{Z_\epsilon^j}(\cdot)$ for $j = \Lambda, \lambda$.

   - *Higher nitpicking costs:*

   $$\pi^K(p_K^{wc}, d_K^{wc}, \epsilon_K^{wc}) \leq \pi^\kappa(p_\kappa^{wc}, d_\kappa^{wc}, \epsilon_\kappa^{wc}),$$

   where $\pi^j(p_j^{wc}, d_j^{wc}, \epsilon_j^{wc})$ is the insurer's equilibrium profit given $c^j(\cdot)$ for $j = K, k$.

2. *When the insurer can commit to a nitpicky strategy, its equilibrium expected profit is unaffected by either factor.*

Proof: The proof is provided in Appendix G. ■

First, when the indemnity cut Z_e is more concentrated around its mean—indicating a more stable threshold—the insurer benefits from reduced uncertainty in indemnity payouts. This stability lowers the probability of the policyholder experiencing high utility loss due to elevated nitpicky levels, thereby increasing the policyholder's willingness to purchase insurance. This, in turn, increases the insurer's equilibrium expected profit. Second, higher nitpicking costs lower the insurer's expected profit. These costs constrain the insurer's ability to optimize profit in the absence of commitment and serve as a deterrent to excessive nitpicking. Finally, the policyholder's reservation utility u_0 acts a constraint on the insurer regardless of its ability to commit. A higher u_0 necessitates offering more favorable contract terms, such as lower premiums or reduced deductibles, to meet the policyholder's participation constraint. This adjustment narrowers the insurer's profit margin.

The economic intuition underlying Proposition 5 is as follows: as indicated in Equation (4), the nitpicky level is inversely related to the deductible. To enhance the policyholder's expected utility, the insurer has an incentive to lower the deductible. However, this comes at the cost of a higher nitpicky level, which reduces the policyholder's utility. Thus, the insurer must carefully balance the trade-off between offering a lower deductible to attract the policyholder and mitigating the disutility caused by a higher nitpicky level.

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4.3 Numerical Illustration

We now provide a numerical example to illustrate the theoretical findings. We begin by specifying the functional forms and parameter values used in the analysis. The loss amount L is modeled as a discrete two-valued distribution, where $\Pr(L = \frac{\bar{\ell}}{2}) = \sigma$ and $\Pr(L = \bar{\ell}) = 1 - \sigma$. The nitpicky cost function is given by $c(e) = \eta\bar{\ell}\left(\frac{1}{1-e} - 1\right)$. Additionally, the indemnity cut Z_e is assumed to follow a Beta distribution, $\text{Beta}(\zeta, \iota)$ with $e = \frac{\zeta}{\zeta+\iota}$.[9] The policyholder's utility function follows a Constant Relative Risk Aversion (CRRA) form, expressed as $v(w) = \frac{w^{1-\gamma}}{1-\gamma}$. Table 2 summarizes the parameter values used for calibration. Using these configurations, we computationally solve the optimization problems (2) and

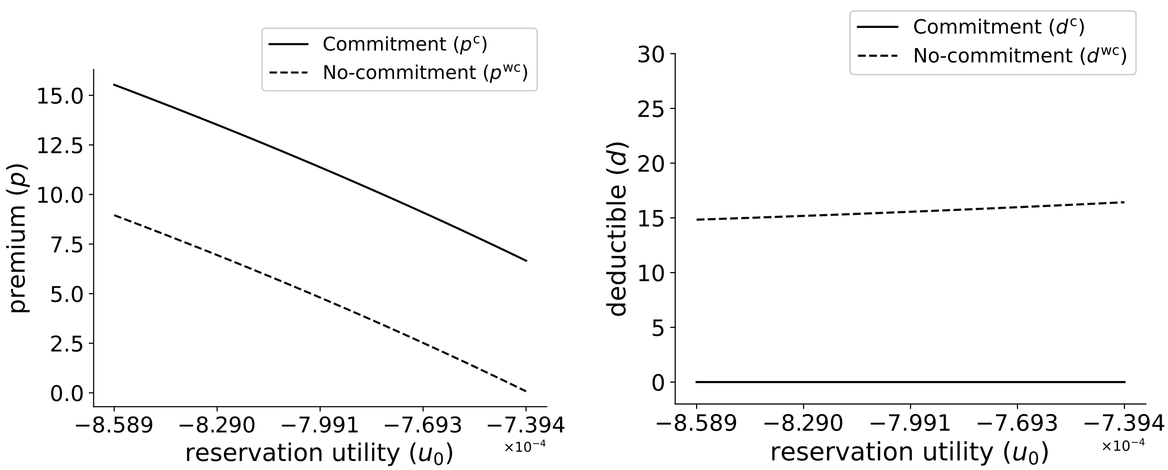
[9] Specifically, $F_{Z_e}(z|e) = \int_0^z \frac{t^{\zeta-1}(1-t)^{\iota-1}}{\mathrm{B}(\zeta,\iota)}dt$ with support on $(0,1)$. With $e = \frac{\zeta}{\zeta+\iota}$, ι is defined as $\iota = \zeta\left(\frac{1}{e} - 1\right)$.

(5) in the case with and without commitment, respectively.

Table 2: Parameters values for numerical illustration

Parameter	q	γ	$\bar{\ell}$	w	η	ζ	σ
Value	0.2	2.5	50	100	0.3	3	0.3

Figure 3 illustrates the optimal premium and deductible under both commitment and non-commitment scenarios, with solid lines representing the commitment case and dashed lines representing the non-commitment case.[10]



(a) optimal premium (p^c/p^{wc})

(b) optimal deductible (d^c/d^{wc})

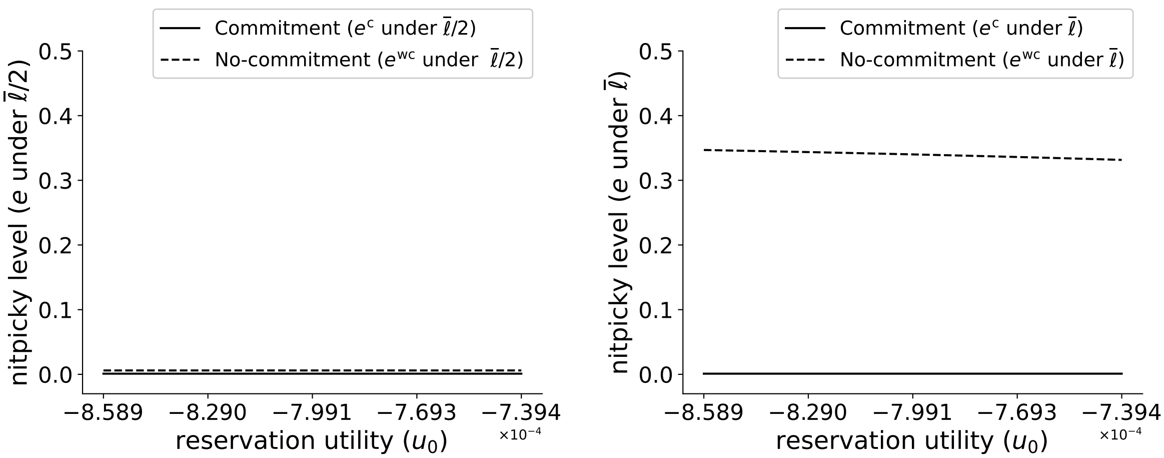
Figure 3: Optimal premium and deductible

In the commitment case, the optimal premium decreases as the reservation utility increases, while the insurance coverage remains full. This result aligns with Proposition 1, which predicts $p^c = w - v^{-1}(u_0)$ and $d^c = 0$. In contrast, under non-commitment, both the premium and the coverage are reduced compared to the commitment case. This confirms Proposition 2, which states that $p^{wc} \leq p^c$ and $d^{wc} \geq d^c$.

Figure 3 displays the optimal nitpicky level under the two scenarios. In the commitment case, nitpicking does not occur regardless of the loss amount, consistent with

[10]The reservation utility u_0 is varied from $(1-\theta)v(w) + \theta\mathrm{E}\big[v(w-L)\big]$ to u_t.

the result in Proposition 1, i.e., $\epsilon^{\mathrm{c}} = 0$ for all ℓ. In contrast, under non-commitment, nitpicking is absent for the small loss ($\ell = \frac{\bar{\ell}}{2}$), but becomes prominent for the larger loss. This observation is consistent with Proposition 2, which predicts positive nitpicky levels for sufficiently large losses.



(a) optimal nitpicky level under $\frac{\bar{\ell}}{2}$ ($e^{\mathrm{c}}/e^{\mathrm{wc}}$) (b) optimal nitpicky level under $\bar{\ell}$ ($e^{\mathrm{c}}/e^{\mathrm{wc}}$)

Figure 4: Optimal nitpicky level under reservation utility change

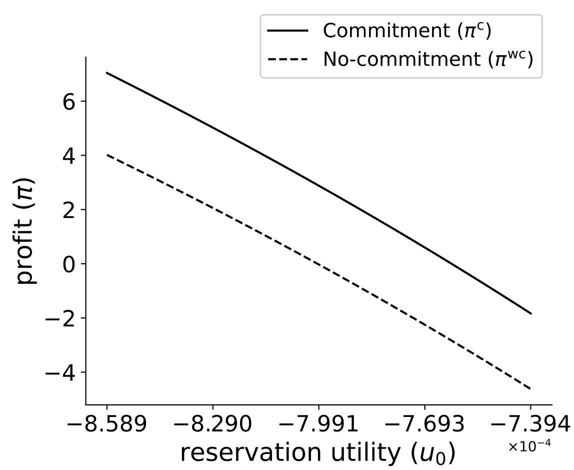


Figure 5: Optimal profit under reservation utility change ($\pi^{\mathrm{c}}/\pi^{\mathrm{wc}}$)

Finally, Figure 5 shows the insurer's optimal optimal profit under both commitment and non-commitment scenarios. The figure shows that inability to commitment reduces the insurer's profitability. This supports Proposition 4. It highlights the economic cost of the lack of commitment, emphasizing the value of a committed nitpicky strategy in insurance markets. Furthermore, the figure illustrates that increasing the reservation utility consistently reduces the insurer's profitability, reflecting the policyholder's greater bargaining power.

5 Extensions

In this section, we extend the results presented earlier to incorporate the scenarios of insurance fraud (Section 5.1) and perfect competition (Section 5.2). These extensions shed light on how the insurer's inability to commit to nitpicky strategies impacts equilibrium outcomes in different contexts.

5.1 Insurance Fraud

Building on the framework established by Picard (1996) and Bourgeon & Picard (2014), this section explores how the inability to commit to a nitpicky strategy influences equilibrium outcomes in the presence of insurance fraud. In their seminal work, Bourgeon & Picard (2014) demonstrated that nitpicking arises alongside insurance fraud when the insurer cannot commit to an auditing strategy. Conversely, both fraud and nitpicking disappear in equilibrium when commitment is possible. However, their analysis assumes that insurers can always commit to a nitpicky strategy. Here, we relax this assumption by allowing the insurer to commit to an auditing strategy but not to a nitpicky strategy. Following this modification, we find that while fraud disappears in equilibrium, nitpicking persists under moderate costs.

To facilitate comparison, this subsection will only consider the scenario in which the insurer *cannot* commit to a nitpicky strategy. We also adopt similar assumptions to those in Bourgeon & Picard (2014), including deterministic loss $L = \ell$. With a deterministic loss amount, the nitpicky strategy degenerates to a nitpicky level. Table 3 summarizes the key differences between the models and their conclusions.

Consider the sequential framework described in Section 2.2, augmented with the possibility of insurance fraud. The policyholder may defraud the insurer by reporting a loss

Table 3: Comparison between Bourgeon & Picard (2014) and this work

	Bourgeon & Picard (2014)	This Section
Panel A. Model Assumptions		
Commitment to audit fraudulent claims	Yes	Yes
Commitment to nitpick honest claims	Yes	No
Panel B. Model Conclusions		
Insurance fraud arises with moderate cost (k)	No	No
Nitpicking arises with moderate cost (c)	No	Yes

when none has occurred, while the insurer can audit with a committed strategy $\beta \in [0,1]$, which may depend on other contract parameters. Following Bourgeon & Picard (2014), we assume that auditing perfectly reveals fraudulent claims but incurs a cost $k > 0$. Fraudulent claims are penalized with a monetary fine $m > 0$, whereas honest claims are subject to an *ex-post* nitpicky level. Figure 6 illustrates the sequence of actions. An associated game tree is presented in Figure A2.

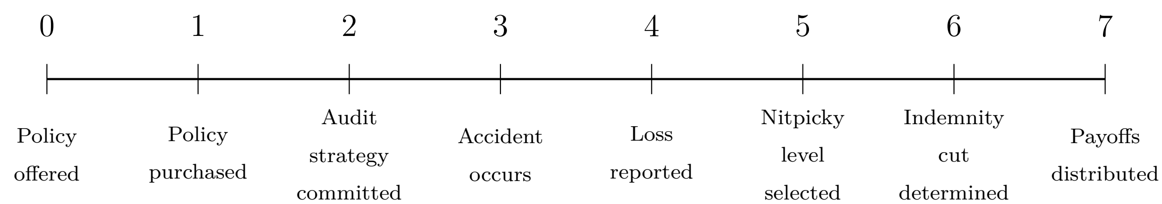


Figure 6: Sequence of play in the presence of insurance fraud.

To avoid repeating the content already presented in Section 2.2, we briefly illustrate the sequence of play as follows. At **Stage 0**, an insurer decides to offer an insurance contract. At **Stage 1**, a policyholder decides to accept the contract. At **Stage 2**, the insurer publicly commits to an audit strategy $\beta \in [0,1]$, which specifies the probability of auditing claims. At **Stage 3**, nature determines whether a loss occurs and, if so, its size ℓ. Subsequently, at **Stage 4**, the policyholder privately decides whether to defraud the insurer by reporting a loss when none has occurred with probability ($\alpha \in [0,1]$). At **Stage 5** the insurer selects an *ex-post* nitpicky level $e \in [0,1]$ given the reported loss. At **Stage**

6, nature determines the indemnity cut. Finally, at **Stage 7**, payoffs are distributed.

The insurer's expected profit and the policyholder's expected utility are given as:

$$\begin{aligned}
\pi(p,d,e,\alpha,\beta) &= p-(1-\theta)\Big\{\alpha\big[(1-\beta)(\ell-d)+\beta k\big]+(1-\alpha)\cdot 0\Big\} \\
&\quad -\theta\Big\{(1-\beta)(\ell-d)+\beta\big[(\ell-d)(1-e)+c(e)+k\big]\Big\}, \\
u(p,d,e,\alpha,\beta) &= (1-\theta)\Big\{\alpha\big[(1-\beta)v(w-p+\ell-d)+\beta v(w-p-m)\big]+(1-\alpha)v(w-p)\Big\} \\
&\quad +\theta\Big\{(1-\beta)v(w-p+\ell-d)+\beta\mathbb{E}\big[v\big(w-p-\ell+(\ell-d)(1-Z_e)\big)\big]\Big\}.
\end{aligned}$$

We solve for the equilibrium contract $(p^{\mathrm{f}}, d^{\mathrm{f}}, e^{\mathrm{f}}, \alpha^{\mathrm{f}}, \beta^{\mathrm{f}})$ by backward induction, as outlined below:

- At **Stage 4 and 5**: If no accident occurred, the policyholder selects the fraud probability α given p, d, and β to maximize her utility:

  $$\alpha^* \in \arg\max_{\alpha\in[0,1]}\Big\{\alpha\big[(1-\beta)v(w-p+\ell-d)+\beta v(w-p-m)\big]+(1-\alpha)v(w-p)\Big\}.$$

  If being reported a loss ℓ, the insurer selects the nitpicky level to maximize profit:

  $$e^* \in \arg\max_{0\le e\le 1}\big\{p-(\ell-d)(1-e)-c(e)-k\big\}.$$

- At **Stage 2**: The insurer commits to an *ex ante* audit strategy β given d and p to maximize its profit:

  $$\beta^* \in \arg\max_{\beta\in[0,1]} \pi(p,d,e,\alpha,\beta),$$

  subject to $e=e^*$ and $\alpha=\alpha^*$.

- At **Stage 0 and 1**: The insurer determines an insurance contract (p, d) to maximize its expected profit, subject to participation and incentive compatibility constraints:

  $$\begin{aligned}
  &\sup_{p,d,e,\alpha,\beta} \pi(p,d,e,\alpha,\beta), \\
  &\text{s.t. } u(p,d,e,\alpha,\beta)\ge u_0, \\
  &\beta=\beta^*,\quad \alpha=\alpha^*,\quad e=e^*.
  \end{aligned}$$

The resulting equilibrium shows that under moderate nitpicky and audit costs, fraud disappears while nitpicking persists when the insurer cannot commit to a nitpicky level. The following proposition formalizes these findings. For notional convenience, we denote:

$$\hat{\beta} = \frac{v(w-p+\ell-d) - v(w-p)}{v(w-p+\ell-d) - v(w-p-m)} \in [0,1).$$

Proposition 6 (Optimal Contract with Insurance Fraud) *When the insurer can commit to an audit strategy but cannot commit to a nitpicky strategy, the optimal contract—assuming its existence and the presence of fraud—is characterized as follows:*

1. *The fraud and audit strategy, α^f and β^f, satisfy:*
   - *if $d^f = \ell$, then $\alpha^f = 0, \beta^f = 0$;*
   - *if $d^f < \ell$, then*
     - $\alpha^f = 0, \beta^f = 1$ *when* $k < (\ell - d^f)e^f - c(e^f)$;
     - $\alpha^f = 0, \beta^f = \hat{\beta}$ *when* $(\ell - d^f)e^f - c(e^f) \leq k < \frac{1-\theta}{\theta}\frac{\ell - d^f}{\hat{\beta}(p^f, d^f)} + (\ell - d^f)e^f - c(e^f)$;
     - $\alpha^f = 1, \beta^f = 0$ *when* $k \geq \frac{1-\theta}{\theta}\frac{\ell - d^f}{\hat{\beta}(p^f, d^f)} + (\ell - d^f)e^f - c(e^f)$.
2. *The nitpicky level e^f satisfies:*
   - *if $d^f = \ell$, then $e^f = 0$;*
   - *if $d^f < \ell$, then*
     - $e^f > 0$ *when* $c'(0) < \ell - d^f$;
     - $e^f = 0$ *when* $c'(0) \geq \ell - d^f$.

Proof: The proof is provided in Appendix H. ■

We briefly explain the economic implications of this proposition. In the absence of insurance (i.e., $d^{\mathrm{f}} = \ell$), there are no incentives for defrauding, auditing, or nitpicking, and consequently, none of these activities occur. However, in case of partial insurance coverage, (i.e., $d^{\mathrm{f}} \in [0, \ell)$), if both nitpicking and auditing costs are not prohibitively high, (i.e. $c'(0) < \ell - d^{\mathrm{f}}$ and $k < \frac{1-\theta}{\theta}\frac{\ell - d^{\mathrm{f}}}{\hat{\beta}(p^{\mathrm{f}}, d^{\mathrm{f}})} + (\ell - d^{\mathrm{f}})e^{\mathrm{f}} - c(e^{\mathrm{f}})$), then it becomes profitable for the insurer to engage in nitpicking honest claims ($e^{\mathrm{f}} > 0$) and auditing potentially fraudulent claims ($\beta^{\mathrm{f}} > 0$). Under these circumstances, the insurer's auditing strategy depends on the auditing cost k: for moderate k, the insurer sets the audit probability to exactly match $\hat{\beta}(p^{\mathrm{f}}, d^{\mathrm{f}})$; for negligible k, the insurer audits all claims. Conversely, if both nitpicking and auditing costs are excessively high, the insurer will forgo both auditing and nitpicking, resulting in unambiguous fraud.

Finally, we highlight the interplay between nitpicking and auditing incentives. Nitpicking increases the insurer's incentive to audit claims. Specifically, in the absence of nitpicking in equilibrium (i.e., $e^{\mathrm{f}} = 0$), the threshold for auditing decreases from $k < \frac{1-\theta}{\theta}\frac{\ell-d^{\mathrm{f}}}{\hat{\beta}(p^{\mathrm{f}},d^{\mathrm{f}})} + (\ell - d^{\mathrm{f}})e^{\mathrm{f}} - c(e^{\mathrm{f}})$ to $k < \frac{1-\theta}{\theta}\frac{\ell-d^{\mathrm{f}}}{\hat{\beta}(p^{\mathrm{f}},d^{\mathrm{f}})}$. This demonstrates that nitpicking raises the effective cost threshold for auditing, thereby reinforcing the insurer's willingness to audit claims.

5.2 Perfect Competition

This subsection discusses the optimal contract in the context of perfect competition. In particular, our analysis shows that the main insights of the previous analysis in a monopoly market are preserved. under perfect competition, free entry ensures that insurers earn zero expected profit, while policyholders are free to select contracts that maximize their utilities. Similar to Section 5.1, we assume a deterministic loss amount $L = \ell$ and thus the nitpicky strategy degenerates to a single nitpick level when ℓ is reported.[11]

Let $(p^{\mathrm{pc}}, d^{\mathrm{pc}}, e^{\mathrm{pc}})$ and $(p^{\mathrm{pwc}}, d^{\mathrm{pwc}}, e^{\mathrm{pwc}})$ represent the optimal contracts under perfect competition with and without commitment, respectively. These contracts are characterized as solutions to the following optimization problems:

$$\begin{aligned} &\sup_{p,d,e} \; u(p,d,e) && (7)\\ \text{s.t.}\quad & \pi(p,d,e) \geq 0, \\ & e \text{ is subject to Equation (3) if no commitment}, \\ & p \geq 0, \quad 0 \leq d \leq \ell. \end{aligned}$$

[11]The analysis can be extended to a stochastic loss $L \sim F_L(\ell)$ with qualitatively similar results. For simplicity of exposition, this more general case is abstract from the discussion.

The properties of these optimal contracts are summarized in the following proposition.

Proposition 7 (Optimal Contract under perfect competition) *under perfect competition, the optimal contract is characterized as follows:*

1. ***With commitment:*** *If the insurer can commit to a nitpicky strategy, the optimal contract* (p^{pc}, d^{pc}, e^{pc}) *is given by:*

$$e^{pc} = 0, d^{pc} = 0, p^{pc} = \theta\ell.$$

2. ***Without commitment:*** *If the insurer cannot commit to a nitpicky strategy, the optimal contract* $(p^{pwc}, d^{pwc}, e^{pwc})$ *is characterized by the following conditions:*

   (a) *When* $c'(0) < \ell - d^{pwc}$*,* $e^{pwc} > 0$ *and is uniquely defined by the equation:*

$$\ell - d^{pwc} = c'(e^{pwc}),$$

   and the premium p^{pwc} *is uniquely determined by:*

$$p^{pwc} = \theta\big[(\ell - d^{pwc})(1 - e^{pwc}) + c(e^{pwc})\big].$$

   (b) *When* $c'(0) > \ell - d^{pwc}$*, the optimal contract simplifies to:*

$$e^{pwc} = 0, d^{pwc} = 0, p^{pwc} = \theta\ell.$$

Proof: The proof is similar to those of Proposition 1 and 2, and is therefore omitted. ■

The conclusions and economic interpretations of Proposition 7 closely align with those derived from Proposition 1 and 2. However, a key distinction lies in the actuarial fairness of the premium under perfect competition. Here, regardless of whether commitment is present, the premium is actuarially fair due to the competitive market structure, which drives the insurer's expected profit to zero. Furthermore, the previous analysis of Pareto optimality, social welfare, and comparative statics can be similarly applied and lead to analogous conclusions.

6 Conclusion

This paper sheds light on the pervasive issue of nitpicking in insurance markets, demonstrating its roots in the insurer's commitment problem. Our analysis shows that when insurers can credibly commit to predefined nitpicky strategies, nitpicking behaviors vanish and full insurance coverage is reached in equilibrium. In contrast, in the absence of commitment, nitpicking emerges as an *a posteriori* profit-maximizing strategy, leading to Pareto-inefficient contracts and diminished social welfare.

From a policy perspective, these findings underline the importance of enhancing commitment mechanisms in insurance markets. Regulators could enforce greater contract clarity by standardizing terms and explicitly defining claim-handling procedures, reducing opportunities for discretionary practices. Technological advancements, such as blockchain and smart contracts, present promising solutions to automate claims processing, ensuring adherence to pre-agreed terms. Additionally, increased regulatory oversight, including audits of claim-handling patterns, and penalties for excessive claim reductions or delays, could deter nitpicky practices.

The results also highlight the broader implications of the commitment problem in insurance markets. These results are important because the resolution of nitpicking behaviors can restore trust, improve efficiency, and ensure the industry fulfills its fundamental role of mitigating risk and uncertainty. Future research could further explore the interplay between technological innovation and regulatory policies in addressing commitment issues, providing a road map for fairer and more efficient insurance markets.

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A Figures

Figure A1 presents a game tree associated with Figure 1. We use Ⓘ, Ⓟ, and Ⓝ to represent the "insurer", "policyholder", and "nature".

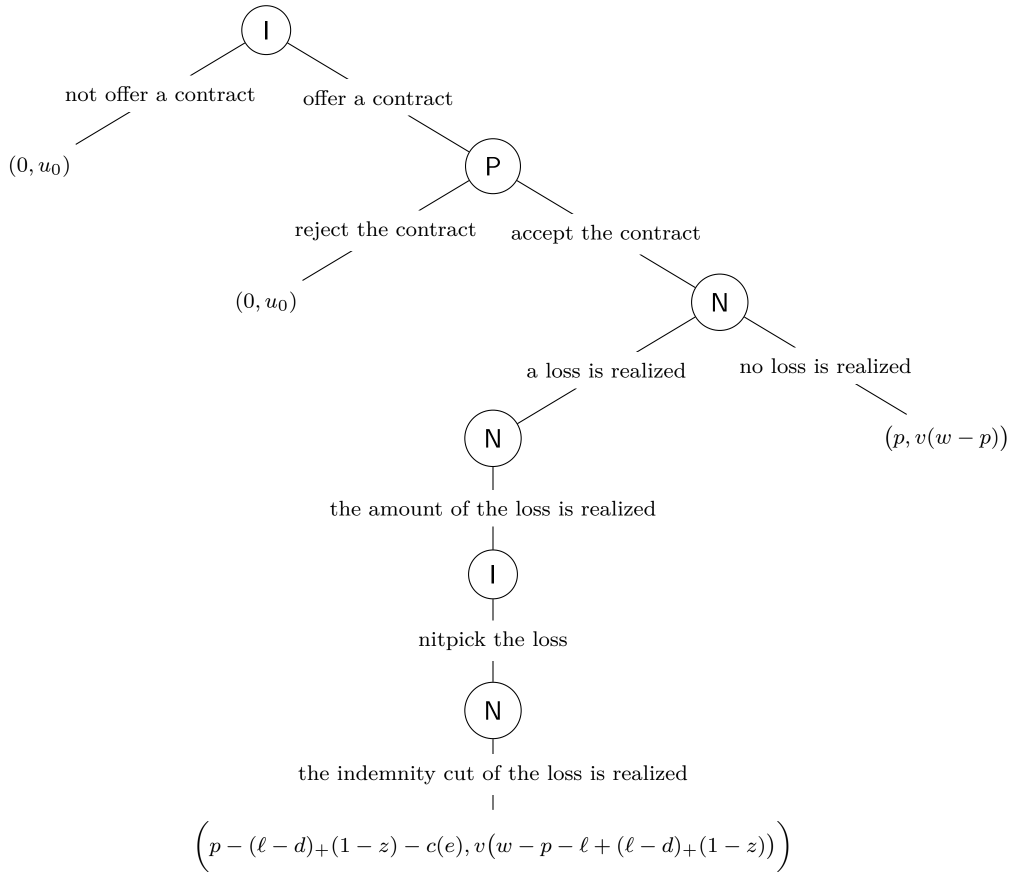

Figure A1: Game tree with an insurer and a policyholder.

Note: At each terminal node, the left element represents the realized profit of the insurer, and the right element represents the realized utility of the policyholder.

Figure A2 presents a game tree associated with Figure 6.

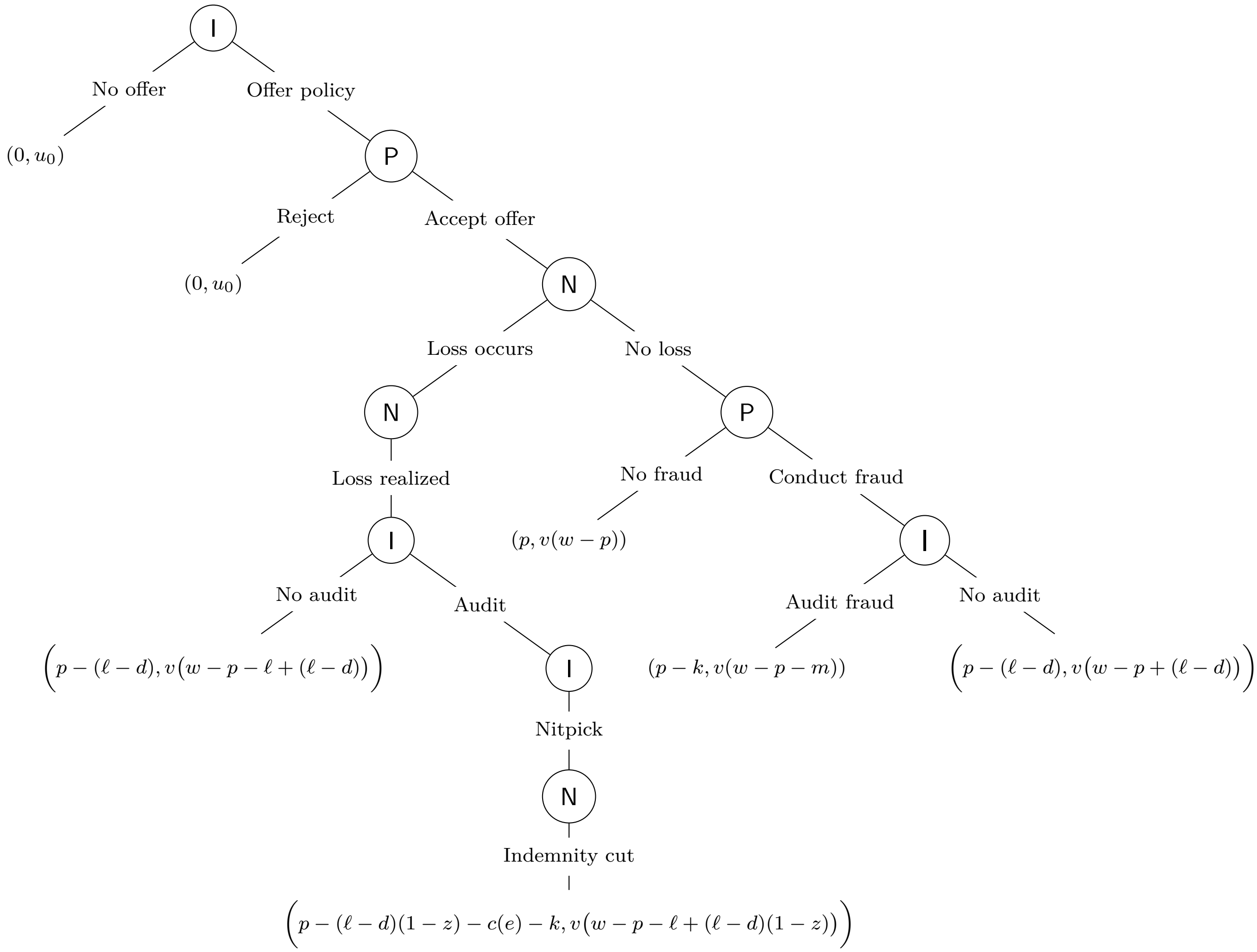

Figure A2: Game tree with an insurer and a policyholder in the presence of insurance fraud.

Note: At each terminal node, the left element represents the insurer's realized profit, and the right element represents the policyholder's realized utility.

B Proof of Proposition 1

Step 1: Proving infeasibility when $u_0 > v(w)$

When $u_0 > v(w)$, we have:

$$u(p,d,\epsilon) \leq u(p=0,d=0,e=0) = v(w) < u_0, \quad \forall \ell \in (0,\bar{\ell}].$$

This implies $u(p,d,\epsilon) < u_0$ for all p, d, and ϵ, indicating that no feasible solutions exist.

Step 2: Proving $\epsilon^{\mathrm{c}} = 0$ when $u_0 \leq v(w)$

When $u_0 \leq v(w)$, the insurer's profit function is given by:

$$\begin{aligned} \pi(p,d,\epsilon) &= p - q\int_0^{\bar{\ell}} \big[(\ell-d)_+(1-\epsilon)+c(\epsilon)\big]\,dF_L(\ell) \\ &= p - q\,\mathbb{E}\big[(L-d)_+(1-\epsilon)+c(\epsilon)\big]. \end{aligned} \tag{B.1}$$

We will now prove $\epsilon^{\mathrm{c}} = 0$ for all ℓ by the method of contradiction.

Step 2.1: Constructing an alternative contract

To begin with, suppose that there exists a first-best equilibrium contract, $(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}})$, with $\epsilon^{\mathrm{c}} > 0$ for some ℓ with strictly positive probability. Next, consider an alternative contract $(p^{\mathrm{A}}, d^{\mathrm{A}}, \epsilon^{\mathrm{A}})$, where $p^{\mathrm{A}} = p^{\mathrm{c}}$, $\epsilon^{\mathrm{A}} = 0$ for all realizations $\ell \in (0,\bar{\ell}]$, and d^{A} is defined as the solution to the following equation:

$$\mathbb{E}\big[(L-d^{\mathrm{A}})_+\big] = \mathbb{E}\big[(L-d^{\mathrm{c}})_+(1-\epsilon^{\mathrm{c}})\big]. \tag{B.2}$$

We assert that $d^{\mathrm{A}} > d^{\mathrm{c}}$. If this is not the case, then, since $\epsilon^{\mathrm{c}} > 0$ for some ℓ with strictly positive probability, it follows that:

$$\mathbb{E}\big[(L-d^{\mathrm{A}})_+\big] \geq \mathbb{E}\big[(L-d^{\mathrm{c}})_+\big] > \mathbb{E}\big[(L-d^{\mathrm{c}})_+(1-\epsilon^{\mathrm{c}})\big].$$

This contradicts Equation (B.2), so $d^{\mathrm{A}} > d^{\mathrm{c}}$.

Step 2.2: Comparing profits under the alternative contract

The insurer's profit under the alternative contract is greater than the profit with $(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}})$:

$$\pi(p^{\mathrm{A}}, d^{\mathrm{A}}, \epsilon^{\mathrm{A}}) = p^{\mathrm{A}} - q\,\mathbb{E}\big[(L-d^{\mathrm{A}})_+\big]$$

$$\begin{aligned}
&= p^{\mathrm{c}} - q\,\mathbb{E}\big[(L-d^{\mathrm{c}})_+(1-\epsilon^{\mathrm{c}})\big] \\
&> p^{\mathrm{c}} - q\,\mathbb{E}\big[(L-d^{\mathrm{c}})_+(1-\epsilon^{\mathrm{c}}) + c(\epsilon^{\mathrm{c}})\big] \\
&= \pi(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}).
\end{aligned}$$

Step 2.3: Comparing policyholder's expected utility under the alternative contract

The policyholder's expected utility is:

$$\begin{aligned}
u(p,d,\epsilon) &= (1-\theta)v(w-p) + \theta \int_0^{\bar{\ell}} \int_0^1 v\big(w-p-\ell+(\ell-d)_+(1-z)\big)\,dF_{Z_\epsilon}(z)\,dF_L(\ell) \\
&= (1-\theta)v(w-p) + \theta\,\mathbb{E}\big[v\big(w-p-L+(L-d)_+(1-Z_L)\big)\big].
\end{aligned}$$

Under the alternative contract $(p^{\mathrm{A}}, d^{\mathrm{A}}, \epsilon^{\mathrm{A}})$, we have:

$$u(p^{\mathrm{A}}, d^{\mathrm{A}}, \epsilon^{\mathrm{A}}) = (1-\theta)v(w-p^{\mathrm{A}}) + \theta\,\mathbb{E}\big[v\big(w-p^{\mathrm{A}}-L+(L-d^{\mathrm{A}})_+\big)\big]$$

Let $Y^{\mathrm{A}} := L-(L-d^{\mathrm{A}})_+$ and $Y^{\mathrm{c}} := L-(L-d^{\mathrm{c}})_+(1-Z_L)$. By definition:

$$\begin{aligned}
\mathbb{E}[Y^{\mathrm{A}}] &= \mathbb{E}[L] - \mathbb{E}[(L-d^{\mathrm{A}})_+] \\
&= \mathbb{E}[L] - \mathbb{E}\big[(L-d^{\mathrm{c}})_+(1-\epsilon^{\mathrm{c}})\big] \\
&= \mathbb{E}[L] - \mathbb{E}\big[(L-d^{\mathrm{c}})_+\,\mathbb{E}[1-Z_L \mid L]\big] \\
&= \mathbb{E}[L] - \mathbb{E}\big[(L-d^{\mathrm{c}})_+(1-Z_L)\big] \\
&= \mathbb{E}[Y^{\mathrm{c}}].
\end{aligned}$$

Lemma 2 *Let $F_{Y^A}(y) = \Pr(L-(L-d^A)_+ \le y)$ and $F_{Y^c}(y) = \Pr(L-(L-d^c)_+(1-Z_L) \le y)$. There exists a $d^A \in \mathbb{R}$ such that*

$$\begin{aligned}
F_{Y^A}(y) &\le F_{Y^c}(y), \quad y < d^A; \\
F_{Y^A}(y) &\ge F_{Y^c}(y), \quad y > d^A.
\end{aligned}$$

Proof: See Appendix B.1. ■

Combining $\mathbb{E}[Y^{\mathrm{A}}] = \mathbb{E}[Y^{\mathrm{c}}]$, Lemma 2, and the definition of the policyholder's expected utility:

$$u(p,d,\epsilon) = (1-\theta)v(w-p) + \theta \int_0^{\bar{\ell}} \int_0^1 v\big(w-p-\ell+(\ell-d)_+(1-z)\big)dF_{Z_\epsilon}(z)dF_L(\ell)$$

$$= (1-\theta)v(w-p) + \theta \mathrm{E}\Big[v\big(w-p-L+(L-d)_+(1-Z_L)\big)\Big],$$

it holds that $u(p^{\mathrm{A}}, d^{\mathrm{A}}, \epsilon^{\mathrm{A}}) > u(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}})$.

Finally, since the alternative contract $(p^{\mathrm{A}}, d^{\mathrm{A}}, \epsilon^{\mathrm{A}})$ provides higher profit for the insurer and does not reduce the policyholder's utility, the original contract $(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}})$ cannot be optimal if $\epsilon^{\mathrm{c}} > 0$ from some ℓ. Thus, $\epsilon^{\mathrm{c}} = 0$ for all ℓ.

Step 3: Deriving d^{c} and p^{c} when $u_0 \leq v(w)$

We now prove that $d^{\mathrm{c}} = 0$ and $p^{\mathrm{c}} = w - v^{-1}(u_0)$ when $\epsilon^{\mathrm{c}} = 0$ for all $\ell \in (0, \bar{\ell}]$.

Step 3.1: Proving $d^{\mathrm{c}} = 0$

First, assume, by contradiction, that $d^{\mathrm{c}} > 0$. Construct an alternative contract $(p^{\mathrm{B}}, d^{\mathrm{B}}, \epsilon^{\mathrm{B}})$, with $d^{\mathrm{B}} = 0$, $\epsilon^{\mathrm{B}} = 0$ for all $\ell \in (0, \bar{\ell}]$, and p^{B} satisfying the equation:

$$p^{\mathrm{B}} = p^{\mathrm{c}} + q\,\mathbb{E}\big[L - (L-d^{\mathrm{c}})_+\big].$$

By construction, $p^{\mathrm{B}} \geq p^{\mathrm{c}}$. The insurer's profit with the alternative contract is:

$$\begin{aligned}
\pi(p^{\mathrm{B}}, d^{\mathrm{B}}, \epsilon^{\mathrm{B}}) &= p^{\mathrm{B}} - q\,\mathbb{E}[L] \\
&= p^{\mathrm{c}} + q\,\mathbb{E}\big[L - (L-d^{\mathrm{c}})_+\big] - q\,\mathbb{E}[L] \\
&= p^{\mathrm{c}} - q\,\mathbb{E}\big[(L-d^{\mathrm{c}})_+\big] \\
&= \pi(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}).
\end{aligned}$$

Next, we verify that the alternative contract does not reduce the policyholder's utility. The policyholder's utility under the alternative contract is:

$$\begin{aligned}
u(p^{\mathrm{B}}, d^{\mathrm{B}}, \epsilon^{\mathrm{B}}) &= v(w - p^{\mathrm{B}}) \\
&= v\Big(w - p^{\mathrm{c}} - q\,\mathbb{E}\big[L - (L-d^{\mathrm{c}})_+\big]\Big).
\end{aligned}$$

By Jensen's inequality:

$$v\Big(w - p^{\mathrm{c}} - q\,\mathbb{E}\big[L - (L-d^{\mathrm{c}})_+\big]\Big) \geq (1-q)\,v(w-p^{\mathrm{c}}) + q\,v\big(w - p^{\mathrm{c}} - L + (L-d^{\mathrm{c}})_+\big).$$

Thus, it holds that $u(p^{\mathrm{B}}, d^{\mathrm{B}}, \epsilon^{\mathrm{B}}) \geq u(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}})$ for some ℓ. The inequality holds strictly if:

$$\begin{pmatrix} 1-q & q \\ 0 & L - (L-d^{\mathrm{c}})_+ \end{pmatrix}$$

is non-degenerate, which can be verified. Hence, there exists $\iota > 0$ such that:

$$u(p^{\mathrm{B}} + \iota, d^{\mathrm{B}}, \epsilon^{\mathrm{B}}) > u(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}})$$

and:

$$\pi(p^{\mathrm{B}} + \iota, d^{\mathrm{B}}, \epsilon^{\mathrm{B}}) > \pi(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}).$$

This implies that $(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}})$ is not optimal, leading to a contradiction.

Step 3.2: Proving $p^{\mathrm{c}} = w - v^{-1}(u_0)$

Finally, we prove that $p^{\mathrm{c}} = w - v^{-1}(u_0)$, which is equivalent to:

$$u(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}) = u_0,$$

under the conditions $d^{\mathrm{c}} = 0$ and $\epsilon^{\mathrm{c}} = 0$ for all $\ell \in (0, \overline{\ell}]$.

Suppose, for contradiction, that $u(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}) > u_0$. Then, there exists $\iota > 0$ such that:

$$u(p^{\mathrm{c}} + \iota, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}) > u_0,$$

and:

$$\pi(p^{\mathrm{c}} + \iota, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}) > \pi(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}).$$

This contradicts the optimality of $(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}})$. Therefore, $p^{\mathrm{c}} = w - v^{-1}(u_0)$.

B.1 Proof of Lemma A.1

We divide the proof into 3 cases.

Case 1: When $y \leq d^{\mathrm{c}}$, it holds, from $d^{\mathrm{A}} > d^{\mathrm{c}}$, that

$$\begin{aligned} F_{Y^{\mathrm{A}}}(y) &= \Pr(L - (L - d^{\mathrm{A}})_+ \leq y) \\ &= \Pr(L - (L - d^{\mathrm{A}})_+ \leq y, L \leq d^{\mathrm{A}}) + \Pr(L - (L - d^{\mathrm{A}})_+ \leq y, L > d^{\mathrm{A}}) \\ &= \Pr(L \leq y) \\ &= F_L(y) \end{aligned}$$

and

$$F_{Y^{\mathrm{c}}}(y) = \Pr(L - (L - d^{\mathrm{c}})_+(1 - Z_L) \leq y)$$

$$
\begin{aligned}
&= \Pr(L - (L - d^{\mathrm{c}})_+(1 - Z_L) \le y, L \le d^{\mathrm{c}}) + \Pr(L - (L - d^{\mathrm{c}})_+(1 - Z_L) \le y, L > d^{\mathrm{c}}) \\
&= \Pr(L \le y) + \Pr(Z_L \le \frac{y - d^{\mathrm{c}}}{L - d^{\mathrm{c}}}, L > d^{\mathrm{c}}) \\
&\ge F_L(y).
\end{aligned}
$$

This leads to $F_{Y^{\mathrm{c}}}(y) \ge F_{Y^{\mathrm{A}}}(y)$.

Case 2: When $d^{\mathrm{c}} < y < d^{\mathrm{A}}$, it holds, also from $d^{\mathrm{A}} > d^{\mathrm{c}}$, that

$$
\begin{aligned}
F_{Y^{\mathrm{A}}}(y) &= \Pr(L - (L - d^{\mathrm{A}})_+ \le y) \\
&= \Pr(L - (L - d^{\mathrm{A}})_+ \le y, L \le d^{\mathrm{A}}) + \Pr(L - (L - d^{\mathrm{A}})_+ \le y, L > d^{\mathrm{A}}) \\
&= \Pr(L \le y) \\
&= F_L(y)
\end{aligned}
$$

and

$$
\begin{aligned}
F_{Y^{\mathrm{c}}}(y) &= \Pr(L - (L - d^{\mathrm{c}})_+(1 - Z_L) \le y) \\
&= \Pr(L - (L - d^{\mathrm{c}})_+(1 - Z_L) \le y, L \le d^{\mathrm{c}}) + \Pr(L - (L - d^{\mathrm{c}})_+(1 - Z_L) \le y, L > d^{\mathrm{c}}) \\
&= \Pr(L \le d^{\mathrm{c}}) + \Pr(L - (L - d^{\mathrm{c}})_+(1 - Z_L) \le y, L > d^{\mathrm{c}}, Z_L = 0) \\
&+ \Pr(L - (L - d^{\mathrm{c}})_+(1 - Z_L) \le y, L > d^{\mathrm{c}}, Z_L > 0) \\
&= \Pr(L \le d^{\mathrm{c}}) + \Pr(L > d^{\mathrm{c}}, Z_L = 0) + \Pr(d^{\mathrm{c}} < L \le y + \frac{(1 - Z_L)(y - d^{\mathrm{c}})}{Z_L}, Z_L > 0) \\
&\ge \Pr(L \le d^{\mathrm{c}}) + \Pr(d^{\mathrm{c}} < L \le y, Z_L = 0) + \Pr(d^{\mathrm{c}} < L \le y, Z_L > 0) \\
&= \Pr(L \le d^{\mathrm{c}}) + \Pr(d^{\mathrm{c}} < L \le y) \\
&= F_L(y)
\end{aligned}
$$

Therefore, $F_{Y^{\mathrm{c}}}(y) \ge F_{Y^{\mathrm{A}}}(y)$.

Case 3: When $y \ge d^{\mathrm{A}}$, it holds that:

$$
\begin{aligned}
F_{Y^{\mathrm{A}}}(y) &= \Pr(L - (L - d^{\mathrm{A}})_+ \le y) \\
&= \Pr(L - (L - d^{\mathrm{A}})_+ \le y, L \le d^{\mathrm{A}}) + \Pr(L - (L - d^{\mathrm{A}})_+ \le y, L > d^{\mathrm{A}}) \\
&= \Pr(L \le d^{\mathrm{A}}) + \Pr(L > d^{\mathrm{A}}) \\
&= 1
\end{aligned}
$$

Therefore, $F_{Y^{\mathrm{c}}}(y) \le F_{Y^{\mathrm{A}}}(y)$.

C Proof of Lemma 1

We divide the proof into three parts: uniqueness, continuity, and monotonicity.

Step 1. Uniqueness.

Given the loss amount $L = \ell$, the insurer chooses e^* to maximize its profit for $d \in [0, \ell]$:

$$e^* \in \arg\max_{e \in [0,1]} \Big\{ p - (\ell - d)_+ \big(1 - e\big) - c\big(e\big) \Big\}.$$

Hence, the optimal e^* depends only on $d \in [0, \ell]$ but not p. The objective function is continuous with respect to e over the compact set $[0, 1]$, ensuring the existence of at least one maximizer e^{wc} by the Weierstrass theorem (See page 140 of Corbae et al. (2009)).

Let $\pi_1(p, d, e) = p - (\ell - d)_+ \big(1 - e\big) - c\big(e\big)$. Assume, by contradiction, that there exist two distinct maximizers $e_1 \neq e_2$ such that both yield the same maximum profit:

$$\pi_1(p, d, e_1) = \pi_1(p, d, e_2).$$

Define a convex combination of these maximizers as:

$$e_3 := \lambda e_1 + (1 - \lambda) e_2, \quad \text{for } \lambda \in (0, 1).$$

By the strict concavity of the objective function, we have:

$$\pi_1(p, d, e_3) > \lambda \pi_1(p, d, e_1) + (1 - \lambda) \pi(p, d, e_2),$$

which leads to:

$$\pi_1(p, d, e_3) > \pi_1(p, d, e_1) = \pi_1(p, d, e_2).$$

This contradiction shows that $e_1 = e_2$, and thus e^* is the unique maximizer.

Step 2. Continuity.

We already know the following three results: first, the decision variable e belongs to the non-empty compact set $[0, 1]$; second, the objective function is continuous with respect to $(e, d) \in [0, 1] \times [0, \ell]$; third, from the uniqueness result, e^{wc} is the unique maximizer of $\pi_1(p, d, e)$. Based on these three results and by the maximum theorem (see pages 149-151 of Corbae et al. (2009)), e^{wc} is continuous with respect to $d \in [0, \ell]$.

Step 3. Monotonicity.

From the definition of e^*, we can conclude:

$$\epsilon^* = \begin{cases} 0, & \text{if } (\ell - d)_+ \leq c'(0), \\ > 0 \text{ and satisfies } c'(\epsilon^*) = (\ell - d)_+, & \text{if } c'(0) < (\ell - d)_+ \leq c'(1), \\ 1, & \text{if } (\ell - d)_+ > c'(1). \end{cases}$$

We analyze the monotonicity in three cases:

1. When $(\ell - d)_+ \leq c'(0)$, $\epsilon^* = 0$ is a constant, making it trivially non-increasing.

2. When $c'(0) < (\ell - d)_+ \leq c'(1)$, $c'(\epsilon^*) = (\ell - d)_+$. Using the implicit function theorem, we deduce that ϵ^* is strictly decreasing with respect to d.

3. When $(\ell - d)_+ > c'(1)$, $\epsilon^* = 1$ is a constant, which is monotonic non-increasing.

Combining these cases, ϵ^* is monotonic non-increasing over $d \in [0, \ell]$.

D Proof of Proposition 2

We analyze two cases: $u_0 > u_t$ and $u_0 \leq u_t$.

Case 1: $u_0 > u_t$

By the definition of u_t, we have:

$$u(p, d, \epsilon) \leq u_t < u_0,$$

which implies $u(p, d, \epsilon) < u_0$ for all p, d, and ϵ. Therefore, no feasible solutions exist in this case.

Case 2: $u_0 \leq u_t$

In this case, we solve for ϵ^{wc} based on the following Equation (4):

$$\epsilon^{\text{wc}} = \begin{cases} 0, & \text{if } (\ell - d^{\text{wc}})_+ \leq c'(0), \\ > 0 \text{ and satisfies } c'\left(\epsilon^{\text{wc}}\right) = (\ell - d^{\text{wc}})_+, & \text{if } c'(0) < (\ell - d^{\text{wc}})_+ \leq c'(1), \\ 1, & \text{if } (\ell - d^{\text{wc}})_+ > c'(1). \end{cases}$$

We now analyze two subcases depending on the value of $c'(0)$ relative to $\bar{\ell} - d^{\text{wc}}$.

Subcase 2.1: $c'(0) < \bar{\ell} - d^{\mathbf{wc}}$

Assume, by contradiction, that $\epsilon^{\text{wc}} = 0$ for all $\ell \in (0, \bar{\ell}]$. From the solution of ϵ^{wc}, this implies:

$$(\ell - d^{\text{wc}})_+ \leq c'(0), \quad \text{for all } \ell \in (0, \bar{\ell}].$$

In particular, this would imply:

$$\bar{\ell} - d^{\text{wc}} \leq c'(0),$$

which contradicts the assumption $c'(0) < \bar{\ell} - d^{\text{wc}}$. Therefore, $\epsilon^{\text{wc}} > 0$ for some ℓ, let $\hat{\ell} \in (0, \bar{\ell}]$ be the smallest element of these ℓ, then, it follows that $c'(0) < (\ell - d)_+$ for all $\ell \in [\hat{\ell}, \bar{\ell}]$. By the solution of ϵ^{wc}, we conclude that $\epsilon^{\text{wc}} > 0$ for all $\ell \in [\hat{\ell}, \bar{\ell}]$.

Subcase 2.2: $c'(0) \geq \bar{\ell} - d^{\mathbf{wc}}$

In this scenario, by the solution of ϵ^{wc}, we have:

$$\epsilon^{\text{wc}} = 0, \quad \text{for all } \ell \in (0, \bar{\ell}],$$

because $(\ell - d^{\text{wc}})_+ \leq c'(0)$ for all ℓ. This completes the proof.

E Proof of Proposition 3

We analyze two cases based on the relationship between $c'(0)$ and $\bar{\ell} - d^{\text{wc}}$.

Case 1: $c'(0) \geq \bar{\ell} - d^{\mathbf{wc}}$

In this case, from the proofs of Propositions 1 and 2, we know that:

$$p^{\text{j}} = w - v^{-1}(u_0), \quad d^{\text{j}} = 0, \quad \epsilon^{\text{j}}(\ell) = 0, \quad \text{for all } \ell \in (0, \bar{\ell}], \quad \text{j} = \text{c}, \text{wc}.$$

To prove Pareto efficiency, we must show that there is no allocation $(p^{\text{C}}, d^{\text{C}}, \epsilon^{\text{C}})$ such that:

$$u(p^{\text{C}}, d^{\text{C}}, \epsilon^{\text{C}}) \geq u(p^{\text{j}}, d^{\text{j}}, \epsilon^{\text{j}}) \quad \text{and} \quad \pi(p^{\text{C}}, d^{\text{C}}, \epsilon^{\text{C}}) \geq \pi(p^{\text{j}}, d^{\text{j}}, \epsilon^{\text{j}}),$$

with at least one strict inequality.

Assume, by contradiction, that such an allocation $(p^{\text{C}}, d^{\text{C}}, \epsilon^{\text{C}})$ exists, then:

1. If $\pi(p^{\mathrm{C}}, d^{\mathrm{C}}, \epsilon^{\mathrm{C}}) > \pi(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}})$, this would contradict the optimality of $(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}})$.

2. If $u(p^{\mathrm{C}}, d^{\mathrm{C}}, \epsilon^{\mathrm{C}}) > u(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}})$, then there exists an $\iota > 0$ such that:

$$u(p^{\mathrm{C}} + \iota, d^{\mathrm{C}}, \epsilon^{\mathrm{C}}) > u(p^{\mathrm{C}}, d^{\mathrm{C}}, \epsilon^{\mathrm{C}}) \geq u_0,$$

and:

$$\pi(p^{\mathrm{C}} + \iota, d^{\mathrm{C}}, \epsilon^{\mathrm{C}}) > \pi(p^{\mathrm{C}}, d^{\mathrm{C}}, \epsilon^{\mathrm{C}}) \geq \pi(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}}).$$

This also contradicts the optimality of $(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}})$.

Thus, no such allocation exists, and $(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}})$ is Pareto efficient under both scenarios where the insurer can or cannot commit to a nitpicky strategy.

Case 2: $c'(0) < \bar{\ell} - d^{\mathbf{wc}}$

In this case, from Propositions 1 and 2, we know that:

$$p^{\mathrm{c}} = w - v^{-1}(u_0), \quad d^{\mathrm{c}} = 0, \quad \epsilon^{\mathrm{C}} = 0, \quad \text{for all } \ell \in (0, \bar{\ell}].$$

Using the same reasoning as in Case 1, we can infer that $(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{C}})$ is Pareto efficient.

However, when the insurer cannot commit, $(p^{\mathrm{wc}}, d^{\mathrm{wc}}, \epsilon^{\mathrm{wc}})$ with $\epsilon^{\mathrm{wc}} > 0$ for some ℓ is Pareto inefficient. From the proof of Proposition 1, we can construct a Pareto-improving allocation, $(p^{\mathrm{A}}, d^{\mathrm{A}}, \epsilon^{\mathrm{A}})$, satisfying:

$$u(p^{\mathrm{A}}, d^{\mathrm{A}}, \epsilon^{\mathrm{A}}) \geq u_0 \quad \text{and} \quad \pi(p^{\mathrm{A}}, d^{\mathrm{A}}, \epsilon^{\mathrm{A}}) > \pi(p^{\mathrm{wc}}, d^{\mathrm{wc}}, \epsilon^{\mathrm{wc}}).$$

This establishes that $(p^{\mathrm{wc}}, d^{\mathrm{wc}}, \epsilon^{\mathrm{wc}})$ is not Pareto efficient.

F Proof of Proposition 4

Step 1: Policyholder's Expected Utility

Since $u_0 \leq u_t$, there exists an equilibrium contract $(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}})$ for $\mathrm{j} = \mathrm{c}, \mathrm{wc}$. From the participation constraint, it holds that:

$$u(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}}) \geq u_0 = (1 - \theta)v(w) + \theta v(w - L), \quad \text{for } \mathrm{j} = \mathrm{c}, \mathrm{wc}.$$

We now show that:

$$u(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}}) = u_0, \quad \text{for j} = \text{c}, \text{wc}.$$

Proof by contradiction: Suppose $u(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}}) > u_0$. Then, there exists $\iota > 0$ such that:

$$u(p^{\mathrm{j}} + \iota, d^{\mathrm{j}}, \epsilon^{\mathrm{j}}) > u_0,$$

and:

$$\pi(p^{\mathrm{j}} + \iota, d^{\mathrm{j}}, \epsilon^{\mathrm{j}}) > \pi(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}}).$$

This contradicts the optimality of $(p^{\mathrm{j}}, d^{\mathrm{j}}, \epsilon^{\mathrm{j}})$. Therefore, the policyholder's expected utility is equal to their reservation utility:

$$u(p^{\mathrm{wc}}, d^{\mathrm{wc}}, \epsilon^{\mathrm{wc}}) = u(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{C}}) = u_0 = (1-\theta)v(w) + \theta \mathrm{E}[v(w-L)].$$

Step 2: Insurer's Profit

From the optimization problem with commitment (see Equation (2)), we have:

$$\pi(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}) \in \left\{ \sup_{p,d,\epsilon} \; \pi(p, d, \epsilon) : u(p, d, \epsilon) \geq u_0 \right\}.$$

When the insurer cannot commit, we have from Equation (5) that:

$$\pi(p^{\mathrm{wc}}, d^{\mathrm{wc}}, \epsilon^{\mathrm{wc}}) \in \left\{ \sup_{p,d,\epsilon} \; \pi(p, d, \epsilon) : u(p, d, \epsilon) \geq u_0, \; \epsilon^{\mathrm{wc}} \text{ follows Equation (4)} \right\}.$$

Since there is an additional constraint for ϵ^{wc} in the case of no commitment, we conclude that:

$$\pi(p^{\mathrm{wc}}, d^{\mathrm{wc}}, \epsilon^{\mathrm{wc}}) \leq \pi(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}).$$

Step 3: Strict Inequality for Profit

We now show that:

$$\pi(p^{\mathrm{wc}}, d^{\mathrm{wc}}, \epsilon^{\mathrm{wc}}) < \pi(p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}) \quad \text{if and only if } c'(0) < \overline{\ell} - d^{\mathrm{wc}}.$$

"If" part: Assume $c'(0) \geq \overline{\ell} - d^{\mathrm{wc}}$. From Propositions 1 and 2, we know:

$$(p^{\mathrm{wc}}, d^{\mathrm{wc}}, \epsilon^{\mathrm{wc}}) = (p^{\mathrm{c}}, d^{\mathrm{c}}, \epsilon^{\mathrm{c}}).$$

Thus:

$$\pi(p^{\text{wc}}, d^{\text{wc}}, \epsilon^{\text{wc}}) = \pi(p^{\text{c}}, d^{\text{c}}, \epsilon^{\text{c}}).$$

"Only if" part: Assume $c'(0) < \overline{\ell} - d^{\text{wc}}$. If:

$$\pi(p^{\text{wc}}, d^{\text{wc}}, \epsilon^{\text{wc}}) \geq \pi(p^{\text{c}}, d^{\text{c}}, \epsilon^{\text{c}}),$$

then equality must hold:

$$\pi(p^{\text{wc}}, d^{\text{wc}}, \epsilon^{\text{wc}}) = \pi(p^{\text{c}}, d^{\text{c}}, \epsilon^{\text{c}}).$$

However, when the insurer can commit, the equilibrium contract $(p^{\text{c}}, d^{\text{c}}, \epsilon^{\text{c}})$ is uniquely optimal. Therefore, the existence of another contract $(p^{\text{wc}}, d^{\text{wc}}, \epsilon^{\text{wc}})$ achieving the same maximum would contradict uniqueness. Hence:

$$\pi(p^{\text{wc}}, d^{\text{wc}}, \epsilon^{\text{wc}}) < \pi(p^{\text{c}}, d^{\text{c}}, \epsilon^{\text{c}}).$$

G Proof of Proposition 5

We first prove part (1) of the proposition, considering the case where the insurer cannot commit to a nitpicky strategy.

Step 1: Impact of Increased Volatility of the Indemnity Cut

Let $F_{Z_\epsilon^\Lambda}(z)$ and $F_{Z_\epsilon^\lambda}(z)$ represent two distinct CDFs of the random indemnity cut Z as defined in the proposition. Define the expected utility of the policyholder under the indemnity cut $F_{Z_\epsilon^{\text{j}}}(z)$, for $\text{j} \in \{\Lambda, \lambda\}$, as follows:

$$\begin{aligned} u^{\text{j}}(p, d, \epsilon) &= (1-\theta)v(w-p) + \theta \int_0^{\overline{\ell}} \int_0^1 v\big(w - p - \ell + (\ell - d)_+(1-z)\big)\, dF_{Z_\epsilon^{\text{j}}}(z)\, dF_L(\ell) \\ &= (1-\theta)v(w-p) + \theta \int_0^{\overline{\ell}} \mathbb{E}\big[v\big(w - p - \ell + (\ell - d)_+(1 - Z_\epsilon^{\text{j}})\big)\big]\, dF_L(\ell). \end{aligned}$$

Since $v''(\cdot) < 0$, the function $v(w - p - \ell + (\ell - d)_+(1 - Z_\epsilon^{\text{j}}))$ is concave with respect to Z_ϵ^{j}. Further, from the second-order stochastic dominance condition $\int_0^t F_{Z_\epsilon^\lambda}(z)\, dz \leq \int_0^t F_{Z_\epsilon^\Lambda}(z)\, dz$, it follows that:

$$\mathbb{E}\big[v\big(w - p - \ell + (\ell - d)_+(1 - Z_{\epsilon^\Lambda})\big)\big] \leq \mathbb{E}\big[v\big(w - p - \ell + (\ell - d)_+(1 - Z_{\epsilon^\lambda})\big)\big].$$

Integrating over ℓ yields:

$$u^{\Lambda}(p, d, \epsilon) \leq u^{\lambda}(p, d, \epsilon), \quad \forall p, d, e.$$

By assumption $u_0 \leq u_t$, the equilibrium contracts $(p_{\Lambda}^{\text{wc}}, d_{\Lambda}^{\text{wc}}, \epsilon_{\Lambda}^{\text{wc}})$ and $(p_{\lambda}^{\text{wc}}, d_{\lambda}^{\text{wc}}, \epsilon_{\lambda}^{\text{wc}})$ both exist. Setting $(p, d, \epsilon) = (p_{\Lambda}^{\text{wc}}, d_{\Lambda}^{\text{wc}}, \epsilon_{\Lambda}^{\text{wc}})$, we find that:

$$u^{\lambda}(p_{\Lambda}^{\text{wc}}, d_{\Lambda}^{\text{wc}}, \epsilon_{\Lambda}^{\text{wc}}) \geq u^{\Lambda}(p_{\Lambda}^{\text{wc}}, d_{\Lambda}^{\text{wc}}, \epsilon_{\Lambda}^{\text{wc}}) \geq u_0,$$

which implies that $(p_{\Lambda}^{\text{wc}}, d_{\Lambda}^{\text{wc}}, \epsilon_{\Lambda}^{\text{wc}}$ is feasible under $F_{Z_e^{\lambda}}(z)$.

Since π^{j} does not depend directly on the distribution of $Z_{\epsilon^{\text{j}}}$, we have:

$$\pi^{\Lambda}(p_{\Lambda}^{\text{wc}}, d_{\Lambda}^{\text{wc}}, \epsilon_{\Lambda}^{\text{wc}}) \leq \pi^{\lambda}(p_{\lambda}^{\text{wc}}, d_{\lambda}^{\text{wc}}, \epsilon_{\lambda}^{\text{wc}}).$$

Step 2: Impact of Higher Nitpicking Costs

Suppose the cost functions $c^K(e) = c^{\kappa}(e) + \tau$, where $\tau \geq 0$. The additional constant cost τ does not affect the optimization problem because it is independent of the nitpicky level. Therefore, the optimal contracts satisfy:

$$(p_K^{\text{wc}}, d_K^{\text{wc}}, \epsilon_K^{\text{wc}}) = (p_{\kappa}^{\text{wc}}, d_{\kappa}^{\text{wc}}, \epsilon_{\kappa}^{\text{wc}}).$$

The equilibrium profits, however, differ due to the additional cost:

$$\pi^K(p_K^{\text{wc}}, d_K^{\text{wc}}, \epsilon_K^{\text{wc}}) = \pi^{\kappa}(p_{\kappa}^{\text{wc}}, d_{\kappa}^{\text{wc}}, \epsilon_{\kappa}^{\text{wc}}) - \theta\tau \leq \pi^{\kappa}(p_{\kappa}^{\text{wc}}, d_{\kappa}^{\text{wc}}, \epsilon_{\kappa}^{\text{wc}}).$$

Step 3: Case with Commitment

Finally, when the insurer can commit to a nitpicky strategy, the indemnity cut distribution $F_{Z_{\epsilon}^{\text{j}}}(z)$ and cost function $c^{\text{j}}(e)$ do not affect the equilibrium because $\epsilon^{\text{c}} = 0$ for all ℓ (see Proposition 1).

H Proof of Proposition 6

We solve this optimization problem backwards.

Solving $\boldsymbol{\epsilon^*}$. By definition, we know ϵ^* is uniquely determined by Equation (4).

Solving $\boldsymbol{\alpha^*}$. We assume that policyholder (insurer) always opts for a lower fraud (audit) probability when indifferent between a lower and a higher one.[12] For notional convenience, we denote:

$$\hat{\beta}(p,d) = \frac{v(w-p+\ell-d) - v(w-p)}{v(w-p+\ell-d) - v(w-p-m)} \in [0,1).$$

Then, by definition of α^*, we can derive:

$$\alpha^* = \begin{cases} 0, \ \beta \geq \hat{\beta}(p,d) \\ 1, \ \beta < \hat{\beta}(p,d) \end{cases}$$

Solving $\boldsymbol{\beta^*}$. We simplify the optimization problem regarding β^* as :

$$\begin{aligned} \beta^* &\in \arg\max_{\beta\in[0,1]} \pi(p,d,\epsilon^*,\alpha^*,\beta) \\ &= \arg\min_{\beta\in[0,1]} \Big\{ (\ell-d)\big((1-\theta)\alpha^*(1-\beta)+\theta\big) - \theta\beta(\ell-d)\epsilon^* \\ &+ \theta\beta c(\epsilon^*) + \beta k\big(\alpha^*(1-\theta)+\theta\big) \Big\}. \end{aligned}$$

Denote

$$\begin{aligned} \phi(\beta) := (\ell-d)\big((1-\theta)\alpha^*(1-\beta)+\theta\big) - \theta\beta(\ell-d)\epsilon^* \\ + \theta\beta c(\epsilon^*) + \beta k\big(\alpha^*(1-\theta)+\theta\big). \end{aligned}$$

Then, $\beta^* \in \arg\min_{\beta\in[0,1]} \phi(\beta)$. Because $\phi(\beta)$ is lower semicontinuous with respect to $\beta \in [0,1]$, we can infer that β^* belongs to a non-empty set. When $d = \ell$, it is obvious that $\beta^* = 0$. Also, by definition, $\alpha(p,d,\beta^*) = 0$.

Next, we discuss the situation that $d \in [0,\ell)$. When $d \in [0,\ell)$, we have $\hat{\beta} \in (0,1)$. Since $\alpha^* = 0$ under $\beta \geq \hat{\beta}$ and $\alpha^* = 1$ under $\beta < \hat{\beta}(p,d)$, β^* can be reformulated to the solution to:

$$\min\Big\{ \min_{\beta\in[0,\hat{\beta})} \phi(\beta), \min_{\beta\in[\hat{\beta},1]} \phi(\beta) \Big\}$$

[12]This assumption largely simplifies the analysis without loss of economic meaning. Without this assumption, for example, we must deal with a whole set (multiple choices) of fraud probabilities instead of a single one whenever the policyholder is indifferent among them. Similar case applies to the insurer. See Picard (1996) for a rigorous proof.

Because $\phi(\beta)$ is linear with respect to β, it follows that the set of potential candidates for β^* is limited to four distinct values: $0, \hat{\beta} - \iota, \hat{\beta}$ and 1 where $\iota \in \big(0, \hat{\beta}\big)$ is a arbitrarily small positive number. Therefore, β^* is the solution to

$$\min\Big\{\phi(0), \phi(\hat{\beta} - \iota), \phi(\hat{\beta}), \phi(1)\Big\}$$

We assert that $\beta^* \neq \hat{\beta} - \iota$ because

$$\begin{aligned}
&\lim_{\iota \to 0} \phi(\hat{\beta} - \iota) \\
&= (\ell - d)\big((1-\theta)(1-\hat{\beta}) + \theta\big) - \theta\hat{\beta}(\ell - d)\epsilon^* + \theta\hat{\beta}c(\epsilon^*) + \hat{\beta}k \\
&> (\ell - d)\theta - \theta\hat{\beta}(\ell - d)\epsilon^* + \theta\hat{\beta}c(\epsilon^*) + \hat{\beta}k\theta \\
&= \phi(\hat{\beta}).
\end{aligned}$$

Now, we only need to consider $0, \hat{\beta}$ and 1:

1. $\beta^* = 0$ if and only if $\phi(0) \leq \min\big\{\phi(\hat{\beta}), \phi(1)\big\}$;
2. $\beta^* = \hat{\beta}$ if and only if $\phi(\hat{\beta}) \leq \phi(1)$ and $\phi(\hat{\beta}) < \phi(0)$;
3. $\beta^* = 1$ if and only if $\phi(1) < \min\big\{\phi(\hat{\beta}), \phi(0)\big\}$.

This is equivalent to

1. $\beta^* = 0$ if and only if $k \geq \frac{1-\theta}{\theta}\frac{\ell - d}{\hat{\beta}} + (\ell - d)\epsilon^* - c(\epsilon^*)$;
2. $\beta^* = \hat{\beta}$ if and only if $(\ell - d)\epsilon^* - c(\epsilon^*) \leq k < \frac{1-\theta}{\theta}\frac{\ell - d}{\hat{\beta}} + (\ell - d)\epsilon^* - c(\epsilon^*)$;
3. $\beta^* = 1$ if and only if $k < (\ell - d)\epsilon^* - c(\epsilon^*)$.

By assumption, we know the there exists a $(p^{\mathrm{f}}, d^{\mathrm{f}})$ which solves:

$$\begin{aligned}
&\sup_{p,d} \ \pi(p, d, \epsilon^*, \alpha^*(p, d, \beta^*(p, d)), \beta^*(p, d)) \\
\text{s.t.} \quad & u(p, d, \epsilon^*, \alpha^*(p, d, \beta^*(p, d)), \beta^*(p, d)) \geq u_0, \\
& p \geq 0, 0 \leq d \leq \ell
\end{aligned} \tag{H.1}$$

Letting $\beta^{\mathrm{f}} = \beta^*(p^{\mathrm{f}}, d^{\mathrm{f}}), \alpha^f = \alpha^*(p^{\mathrm{f}}, d^{\mathrm{f}}, \beta^*(p^{\mathrm{f}}, d^{\mathrm{f}}))$ and $\epsilon^f = \epsilon^*(d^{\mathrm{f}})$, we complete the proof by the definitions of β^*, α^* and ϵ^*.